RADC-TDR-62-540

296 326

25 January 1963

Final Report

DEVELOPMENT OF TECHNIQUES FOR PREDICTION OF SYSTEM EFFECTIVENESS

by

John C. O'Brien

NORTRONICS, A Division of Northrop Corporation
SYSTEMS SUPPORT DEPARTMENT
500 East Orangethorpe Avenue
Anaheim, California

NSS Report No. 2360

AF Contract No. AF 30(602)-2730

Prepared for

Rome Air Development Center
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United States Air Force
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PREFACE

It was the purpose of this effort to investigate and develop a mathematical model capable of predicting the effectiveness of any given system in any given operational environment. The model was expected to be able to account for the influence of all important parameters of hardware and environment, such as reliability, redundancy, man-made and natural environments, etc., and to express the effectiveness of the system in terms of a single figure of merit.

The SPAN (System Performance Analysis) Model developed in the course of the study utilizes both physical and mathematical modeling techniques and is derived from probability and information theory and the energy flow equations of Laplace and Helmholtz. By transforming system and environmental parameters into SPAN language and appropriate effectiveness numbers, the system is simulated by electrical networks. As an example, transmission phenomena are represented as "T" networks in which the values of the series arms represent the lossy transmissions of both energy and information, and the values of the shunt arms represent the transfer ratios. The initial values of resistance are determined by the product of the effectiveness numbers and a constant resistance determined by the simulation equipment used. The total system effectiveness in this type simulation is represented as an output power which can be optimized by varying the resistances of the simulator. The new resistance values, when translated back into real system parameters, then represent the most effective system configuration.

Contained in this report are a series of transforms for the conversion of parameters to SPAN language and effectiveness numbers, and a demonstration of the simulation technique for the case of a hypothetical system.

It is the opinion of RADC that the Contractor has successfully met the original objective of the effort and the feasibility of utilizing the SPAN technique for determining system effectiveness has been demonstrated. It is recognized, however, that the technique should be expanded in scope to take into account items such as human factors, tradeoffs between cost and reliability or other parameters and expansion of real-time solutions to various system problems. A practical evaluation and validation program also should be undertaken.

FOREWORD

The work performed under this contract is a continuation of a study program sponsored by the Systems Support Department, Nortronics Division of Northrop Corporation. This prior study, in progress for several years, was conducted to develop exact measures of system effectiveness. This report summarizes the results of prior research efforts and coordinates them with the findings of the contract study program.

ABSTRACT

This report presents a novel and effective solution to the problem of quantifying system quality. Techniques for prediction of future performance quality are developed in detail. A long-standing need for a satisfactory tool for this purpose has generated a strong incentive to develop an acceptable methodology. The power of the mathematical and physical modeling developed in this study (called SPAN), is due to the use of a comprehensive metric of performance, comprising both energy and "bound" information measures. SPAN's ability to integrate these factors into a consistent performance descriptor is based on well-known laws of potential energy and probability theory. A calculus of system performance dynamics, compatible with the SPAN metric, is directly obtainable from Laplace's and Helmholtz' equations for energy flow, by inserting the information content of the constraining pattern. The feasibility of simulating a system defined in SPAN terminology by an electrical network is demonstrated. Applications of the SPAN methodology to typical problems of command and control systems, such as reliability, maintainability, and accuracy, are shown.

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1.0 INTRODUCTION

The following study is concerned with the investigation, evaluation and development of mathematical models for the prediction of system effectiveness. A survey of the state-of-the-art shows that considerable research on this subject has been performed. An excellent summary of the status of systems analysis is given by Ellis and Ludwig, "Systems Philosophy." The authors state, "Most devised and hybrid systems defy easy comprehension by virtue of inherent detail. In consequence, not one form of analysis, but many are required to describe the internal relations of the system as well as interactions with its environment." It is this situation which has motivated the present study, and its predecessors, to inquire into the possibility of discovering a homogenizing process or integrating function space, with which these forms of analysis can be organized into a single, congruent, comprehensive methodology. Although originally remote, continued research has brought this objective closer every year, through the work of Pontrjagin, Bellman and Kalaba, Chang, Mesarovic, Paynter, Truxal, Pugachev, von Neumann, Shannon, Carnap, and many others who have contributed to the development of special aspects of system integration. However, the pressing demands for immediate help for overtaxed systems engineers justifies an attempt to devise a tool on a pragmatic basis, with the hope that more formal validation will be forthcoming in the future. The methodologies described herein, including Nortronics' development entitled SPAN (System Performance Analysis), have been developed on this basis.

1.1 Statement of the Problem of Prediction of System Effectiveness

1.1.1 Objective - The ability to predict system effectiveness, with respect to a specific mission, and with the degree of accuracy and reliability required for operation and maintenance of contemporary Air Force command and control systems, is the requirement of this study. It implies the existence of sufficient consistency in behavior of a given system to sustain extrapolation beyond its present state. It also presumes the existence of descriptors of performance effectiveness to give adequate definition for quantitative evaluation of future operability. The orderliness, or organizational pattern underlying system complexity must be representable using conventional function theory, so that conversion of philosophical characterizations to numerical measurements can be made readily with familiar mathematical techniques.

1.1.2 State-of-the-Art - At present, the wide diversification in existing types of performance functions and measures, and the extreme complexity of modern command and control networks, have engendered a multitude of mutually incongruent analysis strategies (similar to those of the three blind men who examined the elephant, and proclaimed him to be like "a snake," "a tree", and "a wall," respectively). This heterogeneity tends to defeat the desired objective of system integration, and to preclude coordination into a unified discipline. Prevailing trends toward more intensive specialization in particular areas of professional interest are creating a serious hiatus in basic generalized approaches to a universal methodology of system evaluation. These areas of specialization are becoming highly stratified, so that theoretical mathematicians, who think in terms of set theory and functional spaces, are isolated from problems of system engineering and operations research which are ordinarily couched in the language of topology and

statistics. To some extent, operations analysts also have become isolated from the work of applied physicists and engineers, who need their mathematical insight for a formal discipline of complex system design and integration.

1.1.3 Solution - To bridge the gaps between these specialized areas and levels of analysis, a common meta-language, employing interpretive quantitative translation systems or models can be presumed to be constructable. The translations must have adequate consistency and functional depth to serve as communication channels or filters and use a common "magic" performance quantifying number. There are at least three major levels of logical refinement in an analytical process:

(a) The theoretical or intuitionistic
(b) The design or formalistic
(c) The applications or pragmatic levels (Ref - Foundations of Logic and Math, Carnap).

A minimum of two types of interface models will be needed to interrelate and unify them. The first will be essentially an abstract mathematical model embodying set theoretical principles in applied form; the second will map the techniques of applied functional mathematics on to practical physics and engineering problems. The mapping should be homomorphic, but not necessarily isomorphic. It is conceivably possible to combine both types in one common master model; however, this supermodel may be too difficult to use. Consequently, for expediency and flexibility, the first approach to the solution of unifying system performance will advisably be confined to the development of two complementary models. These will be referred to as the mathematical and the physical models, respectively, in this report. Mutual compatibility between these two models requires a standardized description of performance, with a strong mathematical functional structure, and at the same time, translatable into the physical phenomena which are the ingredients of system performance.

1.1.4 Scope - The scope of the modeling will, for simplicity, be limited to physical systems in the first trial run. Inclusion of economic, psychological, social and physiological aspects of human performance will unnecessarily complicate and probably degrade the quality of the analysis. Consequently, only engineering, physical, and informational phenomena will be studied. Human behavior, insofar as it can be characterized in these terms, will be considered as a normal system element.

1.2 Survey of Types of Approaches

1.2.1 Techniques - Among the approaches devoted to the solution of system problems are many techniques commonly employed in operations research, systems engineering, control engineering, communications engineering, value engineering, management engineering, econometrics, human engineering, physics, and cybernetics. Many versions of both types of models, - the mathematical and the physical, - described above, are utilized. In some cases, intermediate, hybrid or

combination models attempt to bridge both gaps simultaneously and bring the theoretical mathematics directly to the engineer or physicist. Many of them unfortunately lack sufficient scope and breadth of homomorphism for a general model and are usable only in restricted problem areas.

1.2.2 Examples - Examples of such models are:

(a) The connection tensors of Gabriel Kron (Ref - Tensor Analyses of Networks, Gabriel Kron, Wiley 1939)

(b) The complex frequency plane representation used for electronic filter and servo analysis (Ref - Control Engineers' Handbook, J. Truxal, McGraw Hill 1960)

(c) Laplace Transforms (Ref - Control Engineers' Handbook, J. Truxal, McGraw Hill 1960)

(d) Markov processes (Ref - Statistical Decision Functions, A. Wald, Wiley 1950)

(e) The energetic system concepts of Paynter (Ref - Analyses and Design of Engineering Systems, H. Paynter, Technology Press 1960).

1.2.3 Special Approaches - The peculiar characteristics of these approaches depend on the nature of the problems and on the form taken by the input information. Of necessity, their divergent viewpoints and objectives have favored specialized treatments. These are described in the following paragraphs.

1.2.3.1 Operations Research - Operations research is mainly concerned with the evaluation of working systems or concepts with respect to dynamic environments, defined in probabilistic terms. Operations analysts must then consider phenomena which are not so well authenticated as to be measurable completely in deterministic form as causality functions. Their variables often must be described stochastically (by Markov processes, probability distributions, variances, queueing theory, and approximation methods), with a wide choice of error criteria. They utilize digital computers extensively, as physical models.

1.2.3.2 Systems Design - Systems design engineers, in contrast, rely heavily on deterministic laws relating performance to the logic of causality, for extrapolation from mission requirements to specification of equipment performance. Topological flow or block diagrams and geometrical models are popular in this field of analysis.

1.2.3.3 Control Engineering - Control engineers customarily use Laplace Transforms for analysis of the dynamics of feedback systems, with analog computers as physical models of their control systems. Cybernetics has as its purpose the extension of this field to encompass human control systems and operations, and to integrate all control system techniques.

1.2.3.4 Value Engineering, Management Engineering, and Econometrics - Value engineering, management engineering, and econometrics are greatly affected by human motivation which is difficult to measure or quantify. They are often more concerned with materiel inventories than with dynamic energy systems, although there is no sharp division between them. Statistical models, based on maximum likelihood estimates, are very helpful in these technologies. PERT is a familiar dynamic physical model of this brand of system integration, for analysis of material flow and processing.

1.2.3.5 Human Engineering and Biological Research - Human engineers and biological researchers have been unable to obtain complete solutions by such exact analysis methods, and have found that mechanistic models have generally failed to provide the desired insight. In consequence, modeling of this kind has fallen into disrepute among the biological fraternity. However, psychologists are now finding an increasing number of uses for information theory models of human sensing, thinking and communication.

1.2.4 Diversification of Techniques - The following lists, while not complete, illustrate the range of variability of prevailing analytical strategies used in specialized areas of system integration and performance prediction. A distinction between models and techniques is made here by defining a model as a stable configuration of parameters or parametric representations, while a technique is a method of constructing, exercising or evaluating a model.

1.2.4.1 Mathematical Models

Number Systems
Geometries
Differential Equations
Orthogonal Function Sets
Markov Processes
Algebras
Games
Functionals
Dimensional Manifolds
Eigenvalues, Eigenvectors, Eigenfunctions
Lagrange Multipliers
Statistical Distributions
Geometrical Objects
Tensors, Matrices, Vectors
Quaternions
Complex Variables
Energy Conservation Laws
Helmholtz' Equations
Hamilton's Least Action Principles
Hamilton's Canonical Equations
Lagrange's Equations
Probability Spaces
Latin Squares
Signal Spaces
Reliability Models
Laplace, Fourier, Heaviside Transforms
Bessel, Hankel, Legendre, Gamma Functions
Polynomial Sets
Sets, Groups, Rings
Function Spaces
Characteristic Equations

1.2.4.2 Mathematical Techniques

Symbolic Logic
Statistical Inference
Likelihood Estimation
Approximation Methods

- Operational Calculus
- Least Square Error Minimizing
- Steepest Descent Optimization
- Monte Carlo Sampling
- Queueing Analysis
- Variational Calculus
- Dynamic Programming
- Dimensional Analysis
- Relaxation Methods
- Perturbation Methods
- Boundary Constraint Techniques
- Prediction Techniques
- Matrix and Tensor Calculus
- Cybernetics
- Linear Programming
- Group Dynamics
- Psychometrics
- Econometrics
- PERT
- SPAN
- Root Locus Methods
- Statistical Mechanics
- Deductive Logic
- Inductive Logic

1.2.4.3 Physical Models

- Simulators
- Block Diagrams
- Flow Diagrams
- Records
- Graphs, Charts, Tables
- Trainers
- Test Chambers
- Testers
- Instrumentation Systems
- Checkout Systems
- Analog Computers
- Digital Computers
- Digital Programs
- Schematic Circuits
- Test Procedures
- Spectrum Analyzers
- Transfer Function Analyzers
- Codes
- Patterns
- Perceptrons
- Networks
- Network Analyzers
- Filters
- Switching Networks
- Potential Analyzers
- Displays
- Differentiators
- Differential Analyzers
- Electrolytic Tanks
- Mechanical Linkages
- Harmonic Analyzers

1.2.5 Need for Uniform Approach - Empirically derived rules and algorithms, obtained through intuitive induction, with only limited validation, abound in these fields of system performance evaluation. In general, their existence and widespread use, in spite of high error vulnerability demonstrates the acute need for techniques which will overcome the inadequacy of the unaided human mentality. Even when equipped with these tools, system analysts find it difficult to grasp and correlate completely the manifold of intrasystem couplings and interactions which are involved in the coherent behavior of large systems. The intricacies of modern military complexes, such as global surveillance, command and control networks, create an absolute necessity for an analysis methodology with greater integrating potential, i.e., a meta-methodology which will unite and coordinate the individualistic attacks on this problem.

2.0 RATIONALE OF MODELING

The following is quoted from "The Role of Models in Science" by Arturo Rosenblueth and Norbert Wiener: "No substantial part of the universe is so simple that it can be grasped or controlled without abstraction. Abstraction consists in replacing the part of the universe under consideration by a model of a similar but simpler structure. Models, formal or intellectual on the one hand, or material on the other, are thus a central necessity of scientific procedure."

2.1 Mathematical Models

2.1.1 Definitions - A mathematical model treats of an ensemble of elements and of the connections, relations or couplings between them. The couplings are described by the functional rules of organizational logic which may be associative, heuristic, pragmatic, formalistic or intuitionistic in nature. The interrelations or couplings can be single or multiple, and can be homogeneously or heterogeneously distributed throughout the ensemble. If the relations are static, they are defined by postulates; if dynamic, they are represented by operational rules. Sets of sets and ensembles of ensembles comprise models of models, etc., so that a prime requirement is the definition of the kind and degree of complexity or structure of the model. The dynamics of constructiveness can enter into modeling at the intuitionistic level, as part of the process of improving the model characteristics, by strengthening its degree of coherence.

2.1.2 Validity - Mathematical models do not depend on the existence of corresponding physical situations for their validity. They need only to be self-consistent. However, the degree of abstraction of such models from reality is comparable to the difficulty of converting the results to physically realizable conclusions. For example, fundamental filtering theory predicates extensions of signal responses into negative time, but the known world progresses only in the positive time direction. This discrepancy necessitated the development of a theory of realizable filters.

2.1.3 Purpose - The purpose of a model is to enable studying the correspondence between certain aspects of the modeled situation without the interference caused by the presence of other aspects. In effect, the model, like a map, is a form of filter in itself which enhances more desirable types of information and inhibits the transmission of less desirable types. The information which it presents and transfers is always incomplete. The loss of completeness must be economically justified by a gain in some other feature, such as suitability for certain kinds of problems or certain solution techniques, or characteristics of the input information. The weighting factors which make the model more suitable than the original unfiltered description of the system can be derived from the analyst's specialized capabilities. For example, the procedure of extracting square roots by conventional manual arithmetical computation is quite different from that used by a digital computer. The skills and facilities of the analyst and the kind and significance of the information which is the result of his analysis, and the environment which affects the system under consideration, all tend to influence the choice of model employed.

2.1.4 Scope - In a broader sense, environment can be taken as an essential part of the complete system, which channels the total environmental energy into a particular selected pattern of performance called the mission. In this light, the model must be capable of representing both desired or target environment, and the undesired or "inimical" environment satisfactorily. For highest efficiency, the model should represent all of the environment faithfully, at least in the dimensional projections of the general physical situations, which are characterized by the boundary conditions and forcing functions of the human requirements of operation.

2.1.5 Prerequisites - For a practical prediction of system effectiveness the known performance, both of the system and of the environment, must be specified as space-time functions. The accuracy and extent of prediction desired, and the degree and kind of effectiveness (with respect to mission or task performance) in question, also must be specified. It is advisable to assign appropriate analytical weighting factors to performance parameters and functions for guidance in the selection of suitable modeling techniques and computing facilities.

2.1.6 Optional Types of Models - A mathematical model provides a greater degree of abstraction from reality than a physical model. Thus the purity and strength of a limited scope mathematical "state-space" concept can be more easily maintained without degrading compromises with cold hard facts, than in a physical model where considerations of practical influences originating outside the desired scope of analysis can not always be excluded. Conversely, an intermediate physical model is a valuable adjunct for ease of translation from the mathematical model and for maintaining touch with physically realizable situations. Judicious use of such an intermediate model is especially worthwhile in the final stages of design of a complex system, when the flexibility lost by the use of a two-step modeling process is more than compensated for by the ability to examine details of performance minutely and with close correspondence to actual operation. Alternately, in the incipient phases of design and organization of a system, the perspective afforded by an abstract mathematical model, effectively preserved by minimizing the arbitary constraints of equipment design and restricting the choice and use of limiting parameters, is indispensable for successful system integration.

2.1.7 Modeling Logic - The depth of logical consistency of the mathematical model can be established at any of the levels mentioned above, and defined in more detail in the paragraphs that follow.

2.1.7.1 Associative Logic - Associative logic is concerned mainly with incidental scalar quantities without directional nature. In pure mathematics it is representable by statistical set theory and appropriate models. A physical model based on associative logic is the conditioned reflex of a perceptron, which is independent of any a priori causal connectivity.

2.1.7.2 Heuristic Logic - Heuristic logic is single-step sequentially organized, but not necessarily oriented toward any long-range objective. Group theory is a productive source of models for this type of logic. A Markov process is a typical configuration of heuristic logic modeling.

2.1.7.3 Pragmatic or Empirical Logic - Pragmatic or empirical logic has both sequential and directional characteristics, and is therefore vectorial in nature. However, it does not include multistep strategy. Mathematical rings and fields are suitable models for this level of consistency.

2.1.7.4 Formalistic Logic - Formalistic logic follows a directed or vector multistep process, as exemplified by mathematical algebras and topologies. Boolean logic is a well-known case of a formal logic.

2.1.7.5 Intuitionistic Logic - Intuitionistic logic provides a structure and calculus of multidirectional functionals for optimizing formalistic logic functions and for the development of criteria for this purpose. R. Bellman's methods of dynamic programming are excellent examples of applications of intuitionistic logic modeling.

2.2 Physical Models

2.2.1 Definitions - Physical models, which bring the theoretical considerations "down to earth," bridge the gap between the conceptual and the tangible. Their rationale of organization serves as a second information filter to highlight important performance aspects and to suppress others not immediately pertinent. For example, an analog computer model of a servosystem emphasizes time function characteristics, and unites the Laplace transform mathematical model of the system differential equations with the physical variables of voltage, current and power functions of time in the analog circuit, but it ignores the attendant packaging design problems.

Physical models usually include material, motional and energetic forms of representation, and combinations of these, and often use parametric or transformation techniques such as Laplace transforms.

2.2.2 Type of Physical Models

2.2.2.1 Material Models - Material models depend on material position and conservation laws. A quasi-static map of material inventories, with locations and position information, is an example. Mathematically they correspond to scalar fields.

2.2.2.2 Motional Models - Motional models use the first time derivative of mass or material displacement. Electrical or hydraulic flow analogs are examples of this type of physical model. These are compatible with velocity vector field models in pure mathematics.

2.2.2.3 Energy Models - Energy models are based on the energetic parameters and flow laws which are essentially the second derivatives with time, of material or mass position, representing accelerations. Electrical and thermal analog computers, and radiation models, are typical applications of this type of modeling. Their mathematical analogs are found in kinetic and potential energy theory.

2.2.2.4 Combinations of Material, Motional, and Energy Models - In most systems, combinations of these three are of interest. Thus, first, second, and third order servosystem analog computer models are found in common use. (Third derivative time function models have recently attracted some interest because of theories that nonlinear accelerations are linked with the failure mechanisms of materials. This is largely an unexplored area, otherwise.)

2.2.2.5 Functions Containing Spatial Derivatives - Functions containing spatial derivatives (as parameters of gradient, divergence, curl or rotational components, and concentration of scalar fields) have been widely used as field models of electromagnetic potentials, velocities, forces, and radiation. They are also applicable to thermal diffusion and hydrodynamic situations. However, they have tended to remain a nonengineering class of model, in spite of the general nature of the mathematics which underlies their organization.

2.2.2.6 Transformations - Transformations are processes for converting models from given frames of reference with certain parametric designations into new frames, with new variables. They offer particular advantages in formalizing, calculating, and interrelating the system phenomena. Examples are Argand and phase plane representations, conformal mappings, Venn-Euler diagrams, message spaces, tensors and matrices, Riemann surfaces, Spectral Analyses, canonical transformations, contact transformations, Fourier, Mellin, and Rayleigh transforms, etc. These are techniques rather than models, since they convert without changing the scope or meaning of the original model.

2.2.3 Variance - All of these types of models can be specified according to any preferred degree of probability, uncertainty, variance or error tolerance in the quantities chosen. When the variance can be considered negligible, the variables are essentially quantized and become deterministic, otherwise they retain statistical characteristics.

2.2.4 Philosophy - The preceding discussion has been presented to illustrate the philosophy of choice of the models finally selected, and the ground rules for their organization and application. Because of the chaotic state-of-the-art of system integration, the models proposed in this study have been experimentally evolved as combinations of pragmatic and formalistic logic, and are not necessarily entirely validated at a formal level, nor optimized either generally or particularly. However, their effectiveness in providing a quasi-universal basis of system evaluation appears to be considerably greater than that of contemporary modeling methods encountered during this survey.

2.2.5 Practicability - By the nature of their evolution, these "grass roots" models are physically practicable, having a high correlation index with reality, but by the same token, they lack well-established theoretical pedigrees. The choice of suitable techniques has been limited by this fact. The possibility of developing a satisfactory coherent model with sufficient scope might have been quite low, initially, except for the wealth of analysis techniques in limited areas, already in existence. With the help of available science, the task of formulating a working model has been reduced from the formidable one of constructing a completely new concept out of whole

cloth, to the simple one of fitting together certain concepts already in common use. In other words, it is not necessary to invent new elements, but simply to make available elements "jell" as ingredients of a self-consistent or "gestalt" methodology, or, more aptly, to put together the pieces of a jig-saw puzzle.

3.0 UNIVERSAL SYSTEM EFFECTIVENESS MODEL

3.1 General Prerequisites

3.1.1 Universality - Truly universal models of system effectiveness must represent all varieties of elements in a common language, or "state space," and permit deriving unified dynamic algebras or sets of interrelation laws, and enumerable parameters. Physical system elements always include mass, space, time, electric charge, and static or dynamic field couplings. All real parameters are completely definable as combinatorial forms of these basic quantities. The universality of any real basis of measurement can then be established if the parameters used are reducible to some form embodying these dimensionalized basic quantities.

3.1.2 Simplicity - Simplification of any model can be obtained, at a price, by selecting appropriate rationalizing parametric combinations or "kernels." The selection logic may fall into any of the levels previously described. [For example, additive associations of parameters are useful for statistical groupings, which can be considered sets, such as successive peak values of a noisy signal, which are represented by their amplitude probability distribution and density parameters, with respect to time. Formalistic combinations imply causal laws of combination of parameters, in coherent groups (such as applies to the product of voltage and current which gives electrical power). Intuitionistic parametric combinations are more difficult to envision, but can be compared to groups with organizational characteristics which lend themselves to optimization according to some criterion. Convolution integral forms (such as the correlation function, and the entropy concepts in thermodynamics), often take on these characteristics.]

3.1.3 Technical Approach - These logical levels (of rationalization or simplification) can be compared to the mathematical steps of successive differentiation of performance function algebra. This operation gives an increasingly penetrating insight into the functional nature of performance while, at the same time, paying a price by sacrificing information on the constants of integration and boundary values which determine a particular case. Although it is generally desirable to validate a descriptive methodology to the utmost rigor, expediency sometimes makes it necessary to accept and use incompletely validated methods if the need is acute. (The application of Heaviside operational calculus to the solution of differential equations found in electrical engineering, 40 years before its mathematical validity was completely established, is a famous precedent.) Consequently, the organization of a unified system effectiveness analytical calculus which is the object of this study has been undertaken on a pragmatic basis. If found useful to a limited degree without completely rigorous proofs, it will serve its purpose of an interim tool, until time allows perfection of better tools.

3.1.4 Parameters - The degree of logical refinement as defined above employed in the "state" algebra will determine the optimum choice of these simplifying parametric combinations or "kernels" that are to be used as universal "yardsticks" of system performance.

Examples of conventional parametric combinations representing physical quantities are given in Eshbach's "Handbook of Engineering Fundamentals" in the chapter on physical units and standards, which shows a number of dimensional systems embodying several choices of fundamental units. Two of these systems, the dynamic or physical, and the energetic, from this source, are shown below.

Quantity	Dynamic System	Energetical System
Length	L	L
Time	T	T
Velocity	LT^{-1}	LT^{-1}
Mass	M	$EL^{-2}T^{2}$
Force	MLT^{-2}	EL^{-1}
Pressure	$ML^{-1}T^{-2}$	EL^{-3}
Momentum	MLT^{-1}	$EL^{-1}T$
Energy	$ML^{2}T^{-2}$	E
Power	$ML^{2}T^{-3}$	ET^{-1}
Torque	$ML^{2}T^{-2}$	E
Temperature	t	t
Heat	$ML^{2}T^{-2}$	E
Thermal Capacity	$L^{2}T^{-2}t^{-1}$	$L^{2}T^{-2}t^{-1}$
Thermal Conductivity	$MLT^{-3}t^{-1}$	$ELT^{-1}t^{-1}$
Emissivity	$MT^{-3}t^{-1}$	$ET^{-1}t^{-1}$
Entropy	$ML^{2}T^{-2}t^{-1}$	Et^{-1}
Electric Charge	Q	EV^{-1}
Displacement	QL^{-2}	$EV^{-1}L^{-2}$
Electrical Field Intensity	$MQ^{-1}LT^{-2}$	VL^{-1}
Capacitance	$M^{-1}Q^{2}L^{-2}T^{2}$	EV^{-2}
Current	QT^{-1}	$EV^{-1}T^{-1}$
Voltage	$MQ^{-1}L^{2}T^{-2}$	V
Resistance	$MQ^{-2}L^{2}T^{-1}$	$E^{-1}V^{2}T$
Magnetic Flux	$MQ^{-1}L^{2}T^{-1}$	VT
Induction	$MQ^{-1}T^{-1}$	$VL^{-2}T$

where

M = mass
L = length
T = time
t = temperature
Q = electric charge
E = energy
V = voltage

3.2 Basic Common Denominator

3.2.1 Energy, Mass, Time, Length, Charge, Geometry - The necessity for a single integrating metric or "universal solvent" of system behavior has motivated a search for a ready-made or easily constructed "yardstick." Possibly useful quasi-universal measures can be devised from the basic physical units of length, time, mass and electrical charge, and of their products and ratios (such as energy, momentum, etc. as shown on the previous pages). The majority of contemporary analytical methods employ one or more of these parameters, e.g., filter and servo theory customarily express performance in terms of time functions of voltage or current, $e(t)$ and $i(t)$. Other approaches, such as are found in statistical mechanics, are concerned with velocities, mean-free paths, etc., as measures, which are ensconced in dimensional coordinate geometries, giving vector and tensor characteristics to the measures.

An especially suitable coding parameter for performance integration appears to be the energy metric. Its universal character, its convertibility from one form to another (by familiar transformation laws), its conservative nature, and ready detectability and mensuration make it an attractive choice as a descriptor of dynamic performance (the only type of interest in practical systems). However, a precipitate decision without a systematic analysis of the characteristics of an optimized metric must be avoided. It seems probable that the possibility of further simplification similar to that provided by the use of the energy parameter have not been exhausted.

3.2.2 Dynamics of Action - As a result of specialized attacks on performance analysis, there exists a large previously described variety of simplified, limited dimension, analytical projections of the actual phenomena, which have of themselves only limited significance. Like two-dimensional engineering drawings, which often require isometric projections to interpret them, these techniques are information filters. Their efficacy depends on matching the encoding inherent in the complexity of the physical phenomena, with the decoding capabilities of the human observer and analyst. (An example is a television radio signal which is a model of the actual photographic representation of a scene. Its appropriateness for human consumption requires, however, a physical interpretation model, in the form of the television receiver and picture tube. The suitability of the resulting decoding by the observer is unquestioned. There is still something to be desired in its efficiency of spectrum utilization, so that this model has obviously been perfected at the pragmatic and formalistic levels, but not at the intuitionistic level.) An equivalent

of the isometric projection is needed to unify and integrate these special viewpoints, so that the "woods" as well as the "trees" can be seen. The dynamic or time rate functions of system performance can only be developed from such a unified intracompatible model of system behavior.

3.2.3 Definition of Performance and of Performance Measures - "Performance" is defined in Webster's Dictionary as "the execution of the functions required, - effective operation." In a physical system it can be described as the effect or effects of the flow or change of energy in a prescribed pattern. A universal performance metric must include the following elements:

(a) A known energy level
(b) Rates or ratios of energies
(c) Dimensional constraints or boundary conditions (scalars)
(d) Patterns of organization or configurations (vectors).

3.2.4 Concepts of Availability, Effectiveness, Entropy and Information - Availability of performance, as a measure of system effectiveness, is represented either in deterministic form (force, voltage, temperature, etc.) or probabilistic form (entropy, uncertainty, information, variance, etc.). A conversion rule and a set of common transfer laws linking these two forms must be formulated. Also, the performance of data systems, in terms of channel capacity, bandwidth, and error rates, must be reconciled with the energy terminology of power systems, with which they are often associated. This can be done, using the formalism of the laws of energy physics, as is the case with entropy, if uncertainty is equated to availability. It is easy to postulate that uncertainty (probability) is subjective - a human estimate of availability. While human estimation is subject to error, still, in spite of this handicap, this equivalence postulate is defensible.

3.2.4.1 Proof of Equivalence - The general form of Laplace's equations (also Poisson's, Helmholtz's and others) is given below:

$$\nabla^2 E^{\alpha\beta\ldots}_{\mu\nu\ldots} = -K \frac{\partial E^{\alpha\beta\ldots}_{\mu\nu\ldots}}{\partial t} \qquad (1)$$

that is, the availability of the energy E, in its characteristic tensor pattern $E^{\alpha\beta\ldots}_{\mu\nu\ldots}$, is given by its time rate of flow $\frac{\partial E^{\alpha\beta\ldots}_{\mu\nu\ldots}}{\partial t}$, which is proportional to the Laplacian, (the concentration or spatial "lumpiness" of the energy) $\nabla^2 E^{\alpha\beta\ldots}_{\mu\nu\ldots}$, and an undefined constraint -K. The lack of definition of K allows it to be inserted for the concept of "energy impedance." When these quantities are probabilistic, then, by the equivalence postulate, substitution of "H," (information content), for the partial derivative $\frac{\partial E}{\partial t}$, (the negentropy or availability of energy), and W the information bandwidth, for $\frac{1}{K}$, (the energy conductance) produces a form $H = \frac{\nabla^2 E}{K}$

which can be normalized and differentiated to give the channel capacity law for information density:

$$\frac{dH}{dt} = C \le W \log_2 \frac{(P + N)}{N} \quad . \tag{2}$$

The logarithm of the "humanized" probabilistic estimate of the energy "lumpiness," $\frac{(P + N)}{N}$ has been substituted for the Laplacian (which is really an idealized quantized form for the same thing). Converting appropriate probabilities assigned to each scalar quantity, into their information values, (by $H = -\Sigma p \log p$) the transformation of the deterministic or classical physics form of Laplace's law into the information theoretic form can be completed. If desired, the information content of the tensor itself as a pattern, or n-ordered set, can be derived and used in a more generalized nondimensional expression of the law (when dimensional constraints are unimportant, as in heat conduction). (If the energy couplings or relations involve nonlinearities, such as elasticities, these can be treated in a higher degree expression $\nabla^4 E^{\alpha\beta\cdots}_{\mu\nu\cdots}$).

3.2.5 Quantification - Since the laws governing energy flow are dependent on the conservation of energy, they imply that in a closed system, entropy or "bound" information (the information associated with energy) is also conserved. (Compare Ohm's law, and Kirchoff's laws for voltages and currents.) In other words, information, as the quality of energy, can flow in a closed system conservatively and can be accounted for in quantitative terms. As system "effectiveness" is just another way of expressing performance "quality," then it theoretically becomes possible to quantify system effectiveness and calculate its conservation and flow characteristics. If time dependence can be established, predictions of future effectiveness will then become routine computations. A purely quantitative "figure of merit" for prediction can be constructed from the terms of the equation,

$$\frac{\partial E}{\partial t} = \frac{\nabla^2 E}{K} \quad , \tag{3}$$

when the right hand-terms are reduced to scalar complexity numbers. When $\nabla^2 E$ is a vector, the combinatorial order of the vector must be sacrificed for simplicity. Then the identity of individual permutations will be lost, and the identity or "signature" of typical patterns also. For instance, the pattern complexity numbers of a left-hand screw and a right-hand screw will be the same (and similarly for more complex systems).

3.2.5.1 Identification - This surrender of identity, by classifying all elements alike, in exchange for universal quantification seems a justifiable compromise at this point. Loss of identity is seldom critical in artificial system analysis where most systems and subsystems are identifiable by other means. Otherwise, if all of the permutative identity of each pattern is retained, the enormous number of complex patterns possible will make analysis insufferably complicated. However, partial identification of dimensional patterns can be retained optionally during analysis, if desired, by breaking up the modeling treatment into small portions which can be labeled appropriately.

3.2.5.2 Figure of Merit - The compromise just made consists of substituting a number representing the amount of information inherent in the pattern, for the information itself. The tremendous simplification obtained by this expedient makes quantitative complex system integration practical. The figure of merit then consists of the product of an arbitrary unit of energy, $\bar{e}$ in a particular region, with the information value of the environment in which it occurs, H_{ϵ}, representing its availability, A. The factors of the environment will include:

(a) The probability of simultaneous occurrence of other units of energy
(b) The flow constraints
(c) Other empirical efficiency factors which cannot be readily analyzed.

The figure of merit will be called the performance availability unit (pau), and will be related to an arbitrarily chosen unit energy reference state. Relative (differential) availability (to an adjacent region), will consist of the difference between the absolute availabilities of the two regions. Then a flow diagram can readily be constructed, showing the dynamics of system and subsystem performance availabilities, with respect to any given mission environment (considered as a sink or final load).

3.3 System Interrelations

3.3.1 Flow Laws - The benefits of the use of Laplace's equation lie in the addition, to the logical structure of existing static models, of a logical definition of the laws of dynamic informative energy flow, which depends on the system interrelations. The information value of the interrelations (which, by the conservation of information which has been postulated, is all that is now needed for complete analysis of the flow rates) can be obtained either by direct measurement, or indirectly by deduction. (Ohm's law for example, defines resistance as the ratio of voltage to current. Both voltage and current are measurable directly, but resistance is not measurable as such and is defined only parametrically.) The result of this determination will be a "transmittance," "conductance," or "impedance" parameter which represents the effect of the system couplings on informational energy flow (pau).

3.3.2 System Constraints - The constraints on pau flow will usually be defined in the system description. Most artificial systems consist mainly of a number of lumped constraints (although distributed constraints are occasionally encountered, as, for instance, in radiation systems, plasmas, etc.). Direct analysis of the average information content of these constraints is sometimes difficult, although transfer functions and similar formal product laws of behavior can be readily reduced to $-\Sigma p \log p$. The probabilities in question can be normalized easily if the variances are uniform or negligible. Then the information content becomes the sum of the logarithms of the conditional probability steps and of their coefficients or scale factors. For example, the information content of the transfer function

$$\frac{G\,(1 + sT_1)}{(1 + sT_2)} \tag{4}$$

becomes

$$\log G + \log (1 + sT_1) - \log (1 + sT_2) \tag{5}$$

if deterministic, or

$$p_G \log p_G + p_1 \log p_1 + p_2 \log p_2 \tag{6}$$

if probabilistic. Since the information content is a universal metric, its average can be used to define average effectiveness. (Apples and oranges cannot be added, but apple calories and orange calories can be added to get total caloric content. In effect, the information content is the equivalent of the caloric content.) By this means a universally applicable numerical figure of merit can be obtained as a metric coefficient for system effectiveness, which is completely valid and exact. In the case of constraints, the information content is parametric, and represents the effect on informational energy flow with a given availability gradient (similar to bandwidth in the flow of abstract information). Like any other logarithmic measure, it is concerned with ratios and needs an arbitrary reference (such as is used for DBM, DBW, etc.).

3.3.3 System Couplings - The system couplings comprise the paths along which energy can flow between elements in terms of mass, length, time, or generalized availability. Since various forms of energy - electrical, thermal, mechanical, radiative - depend on separate transfer mechanisms for their flow, these couplings will include electrical connections; hydraulic piping; thermally conductive, convective and radiative paths; mechanical linkages; optical beams; magnetic fields; gravitational fields; etc. (A comprehensive analysis of energy flow paths is given by Prof. Henry M. Paynter, of MIT, in his book, Analysis and Design of Engineering Systems.) The index of logical structure of these paths, considered as modifiers or controllers of the energy, is the pattern number which takes the place of "energy impedance (conductance)," in Laplace's equations. In effect, a constraint would represent an "impedance" while a coupling would be termed a "conductance" or "transmittance." Actually, they are reciprocally related and either may be used in the proper sense, to represent the control of energy flow.

3.3.4 Conservation Laws - The laws of control of energy flow are derived from the above-mentioned fundamental conservation laws, which include:

- (a) Conservation of mass
- (b) Conservation of electrical charge
- (c) Conservation of energy - the first law of thermodynamics
- (d) Conservation of entropy, or "bound" information - in reversible systems
- (e) Conservation of "action" - Hamilton's Least Action Principle
- (f) Conservation of variance - a special case of "c."

3.3.5 Configurations, Combinatorial Techniques - The combinations of elements and couplings, or constraints, can be represented as configurations, which in turn can be characterized as deterministic, statistical, or mixed. A Markov process is an example of a mixed configuration, in which the dependence of the beginning of each excursion on the terminal point of the previous excursion is absolutely deterministic, but the dependence of the direction taken by the excursion is only statistically dependent on the preceding one. Methods of representing configurations include topologies, flow graphs, matrices and tensors, etc., and the more general forms of sets, classes, groups, rings, spaces, fields, etc., which categorize the coupling combinations and permutations to some degree. It is obvious that an astronomical number of combinatorial permutations of elements and couplings exist, and that the "signature" or identification of each distinctive permutation is unique in itself.

3.3.6 System Coherence Measures

3.3.6.1 Description of Coherence - Coherence in a mathematical model representing a real physical system can be expressed by continuous functions or logically organized sets which provide the desired number and strength of connecting links between elements. The probabilistic measures which often occur in the description of the connections are sometimes given on a logarithmic scale, which converts the statistically causal dependency functions to simplified additive forms. This is especially useful with nonlinear functions. The need for extending the coherence in the mathematical model to include the physical model restricts the types of logic which describe the coherence, to those which will satisfy both type of models.

3.3.6.2 Nature of Coherence - Coherence is the result of either external repulsive forces, such as constraints due to conservation laws, or of internal forces such as mutual attractions between elements. At present, gravitational attraction appears to be the only known universal association-seeking or attractive force occuring in physical systems. There are some attraction laws depending on a duality or complementary nature of the elements for their operation, such as electrostatic and electromagnetic attraction. Mathematically, it is possible to describe all attractions, single or dual, as the result of external repulsive constraints, or conservation laws.

3.3.6.3 Optimum Coherence Logic - The optimum choice of a causally connective logic for modeling the coherence of a real system will depend on the characteristics which are most important to the analyst. (For example, the voltage peaks on a commercial high power line are of greatest significance to the insulator designer, while the current levels are more important in selecting the optimum conductor size. The former would be most suitably modeled by a voltage conserving logic, such as Kirchoff's law for voltages in a closed loop; the latter would require a current conserving logic, such as Kirchoff's law for the summation of currents at a junction.)

3.3.6.4 Universality of Coherence - The systems analyst is ordinarily concerned with complex interrelationships of subsystem performance parameters and with the techniques of optimizing their combined behavior. He needs both a unified measure of all behavior phenomena, and a universally coherent logic or discipline of organization. An acceptable definition of performance

for this purpose as mentioned above is the product of energy, in a controlled pattern, and its availability, or utility in terms of a given load, or mission. Dynamically speaking, energy change is defined as constituting an energy event, which is the unit of action. Reiterating, the constructive complexity level or entropy of the energy event control pattern, corresponding to the dynamic quality of its performance, then represents its availability, to a contiguous region, by the logically coherent laws of causality governing its transfer.

3.3.6.5 Relativity - Availability is usually relative or differential with respect to a particular application or use. A definition of absolute availability may be written as that of a given performance event with reference to a "ground state" load. Relative availability will then become the difference between two absolute availability levels. All the ramifications of availability such as effectiveness, utility, reliability, readiness, kill probability, efficiency, power factor, etc., are then definable as special cases.

3.3.6.6 Information Content - The definition of performance given above has a close correspondence with the semantic concept of performance given in Webster's dictionary. The extension of this correlation to include the realm of information, as a measure of the quality of availability, is the next logical step. Average information content, (equivalent to entropy), is equal to the function $-\Sigma p \log p$ of the probabilities of performance. The logical dependence of the availabilities of a series of successive energy events, represented as conditional probabilities, becomes a simple sum of the information contents of each event. This sum is the total availability of the combined sequence of performance events. As a time function, it is identically the reliability of the system. This is easily seen if the product of the individual functions of reliability for each sequential stage, $\exp\left(\frac{-t}{m}\right)$, is converted to the logarithmic form, $-\frac{t}{m}$. It also equals the time integral of the objective deterministic energy availability time gradient, $\int_{t_1}^{t_2} \nabla^2 E \, dt$ which equals $\log R$, the reliability. The simplicity of this treatment can be a great advantage in reliability analysis.

3.3.6.7 Conclusion - Thus it appears possible to construct a physical energy model of a system by using for the coherence functional, the causality resulting from the conservation of energy, for predicting future performance conditions. The universal nature of energy makes it a preferred parameter when used as a common denominator of work done (a practical definition of performance).

3.3.7 System Effectiveness, Mission Achievement - Work done, alone, is not a completely satisfactory measure of system effectiveness. The question of whether the work is done in the desired manner on the desired target objective or load must be answered. The disturbing effects of the load must be introduced into the model, together with those of the environmental influences which may affect the quality of the operation. Criteria for determination of acceptable performance at the load must be provided. The couplings between system and load, and between environment and both system and load, must be described and analyzed.

The universal parameter of energy availability is also capable of representing these extra-system factors in a common calculus of system effectiveness. Both load and environmental conditions are amenable to quantification in these units. A standardized analytical method is then permissible for relating all of the factors which control mission achievement.

3.4 System Performance Dynamics - The dynamics of system performance is concerned with the time rates of change of performance in sections of the system. Since performance has been defined as a unit flow of energy qualified by its scale and pattern of availability, the dynamics of conventional energy flow must be extended to include the effects of its availability factor. An examination of the different versions of the energy flow laws, shown below, gives a clue to the modeling technique needed.

3.4.1 Helmholtz Equations

$$\nabla^2 E = -q\,k, \quad \text{Poisson's equation for an electric potential field} \tag{7a}$$

$$\nabla^2 V = \frac{1}{c^2}\,\frac{\partial^2 V}{\partial t^2}, \quad \text{the wave equation, of Helmholtz} \tag{7b}$$

$$\nabla^2 V = \frac{1}{k}\,\frac{\partial V}{\partial t}, \quad \text{the heat diffusion equation of Laplace} \tag{7c}$$

$$\nabla^2 E = \frac{k\mu}{c^2}\,\frac{\partial^2 E}{\partial t^2} + \frac{\sigma\mu}{c^2}\,\frac{\partial E}{\partial t}, \quad \text{the electric conduction equation} \tag{7d}$$

$$\nabla^2 \psi = \frac{8\pi^2}{h^2}(V - E)\,\psi, \quad \text{a reduced form of Schrodinger's equation for wave mechanics} \tag{7e}$$

(E = voltage in the above equations.)

In all of these forms there is a factor on the right-hand side which determines the rate of flow. This factor can be considered to be the result of the "matching" between the original form factor or pattern rank of the energy (a priori information), and the pattern rank of the load region or mission (a posteriori information) into which the performance energy flows. If this factor is constant with time, the time variation of flow rate is equal to the time variation of the original concentration of "lumpiness" and/or the time variation of its pattern or entropy. If the factor is time-dependent, the dynamics involve its variability also, as follows:

$$\text{If} \qquad \nabla^2 E = -k\,\frac{\partial E}{\partial t}, \tag{8}$$

then, if k is a constant,

$$\frac{d}{dt}(\nabla^2 E) = \frac{d}{dt}\left[\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}\right] = \frac{d}{dt}\frac{\partial^2 E}{\partial x^2} + \frac{d}{dt}\frac{\partial^2 E}{\partial y^2} + \frac{d}{dt}\frac{\partial^2 E}{\partial z^2} = -k\frac{\partial^2 E}{\partial t^2} \tag{9}$$

so that the time variability of availability of energy is proportional to the energy acceleration. Similarly: If

$$\nabla^2 E = -k(t)\frac{\partial E}{\partial t} \tag{10}$$

then, if $k(t)$ varies with time,

$$\frac{d}{dt}(\nabla^2 E) = -k(t)\frac{\partial^2 E}{\partial t^2} - \frac{\partial E}{\partial t}\frac{\partial k(t)}{\partial t} \tag{11}$$

The additional term $\frac{-\partial E}{\partial t}\ \frac{\partial k(t)}{\partial t}$ is introduced by the time variation of the transmittance function $k(t)$. This can occur when $k(t)$ is a time function of performance degradation, such as the reliability function $\exp(\frac{-t}{m})$. Then

$$\frac{d}{dt}(\nabla^2 E) = -\exp(\frac{-t}{m})\frac{\partial^2 E}{\partial t^2} + \frac{t}{m}\exp(\frac{-t}{m})\cdot\frac{\partial E}{\partial t} \tag{12}$$

indicates that the nonlinearity of the mean-time-to-failure of a system can cause side order effects of a transient nature in system reliability in addition to the direct influence of the particular failure rate involved.

3.4.1.1 Impedance - A full development of the impedance concept is contained in Morse and Feshback Methods of Theoretical Physics, Chapter 3, pp. 285 et seq., and also in Paynter's Analysis and Design of Engineering Systems. Intrinsically the nature of the impedance concept $k(t)$ is bound up with the mass distribution in the energy flow path since mass acts as a storage and transfer factor for energy. The conservation of mass limits the functional variations which can occur in the transmittal impedance to fairly simple forms. For example, kinetic energy is $\frac{1}{2}mv^2$. If m is a function of t, at any point, its variation can occur only by its translation to another point, by the conservation law of energy and momentum, $T^{ik} = 0$. (T^{ik} is the symmetrical matter tensor. Condon 1-121.) Therefore, its changes become part of the energy equations for that region and are handled compatibly with the translatory energy itself.

When elastic nonlinear constraints are included in the system couplings, the rigid body equations are insufficient. A relationship of the form

$$\nabla^4 E = \phi(t, x, y, z)\frac{\partial^3 E}{\partial t^3} \tag{13}$$

must be employed to completely define the system.

3.4.2 Canonical Equations - The similarity of Helmholtz', Laplace's and Poisson's and Schrodinger's equations is due to their common derivation from Hamilton's equations for the conservation of energy and momentum;

$$\dot{q}_r = \frac{H}{p_r} \text{ , (14a)} \qquad \dot{p}_r = -\frac{H}{q_r} \text{ . (14b)}$$

(In a conservative system Hamiltonian function "H" is energy)

and to Hamilton's least action principle:

$$\delta \int_{t_o}^{t_1} (T - U)\, dt = 0 \tag{15}$$

where T is kinetic energy and U is potential energy.

When the system is not conservative, i.e., when an external force "F_r" is applied, then

$$\dot{q}_r = \frac{H}{p_r} \text{ , (16a)} \qquad \dot{p}_r = -\frac{H}{q_r} + F_r \text{ . (16b)}$$

Motional impedance, "Z_r" analogous to the energy flow impedance mentioned above, is obtained as follows:

$$Z_r = \frac{\dot{p}_r + \frac{\partial H}{\partial q_r}}{\frac{\partial H}{\partial p_r}} \text{ .} \tag{17}$$

3.4.2.1 Identification - Conversion of unwieldy tensor and canonical equations into more manipulable form is accomplished as previously mentioned by dropping the identifiable characteristics which give them the vector character, in favor of the more universal but less significant "information content" of these terms and functions. The resulting quantitative equations and parameters are not restricted to the type or permutative character of any particular organization. The justification for this step is the nondimensional nature of entropy (the quality of energy) which can be expressed either statistically or deterministically. If defined in classical continuous functions, the quality of energy can be reduced to this nondimensional form, as long as the number of discrete or quantized logical steps, required to formulate the functional pattern, can be obtained. The logarithm of these steps, considered as factors, gives an arbitrary coefficient of pattern rank, associated with the actual probability of correctness (or of occurrence) for each quantity or event. Since all measurable variables have some uncertainty, or probability of erroneous definition in their values, the distinction between deterministic and probabilistic functions disappears; or rather, the perfection of continuity of the former is seen to be an illusion. Consequently, the technique of information theory by which the average information content of a complex structure of conditional probabilities can be derived is:

$$H = -\Sigma\, p \log p \text{ ,} \tag{18}$$

(H is average information content) and must be used to obtain the pattern measure of a complex system.

The justification for the step of scuttling the unique signatures of specific patterns of structure and behavior, so as to standardize the descriptions of a class of patterns of equal complexities in a common pattern rank number representing their common information content, is that of pragmatic expediency. From the standpoint of computational facility this step reduces the range of pattern variability from factorials to arithmetic quantities. It is then no longer necessary to distinguish between specific patterns, but only to measure the total number of energy-bits in each.

If it is considered desirable to retain some of the original flavor of identification, it is fairly easy to provide an adequate number of separate models for each dimensional projection which must be identified. This will not ordinarily prohibitively handicap the computations, since most identification characteristics are redundant and only a few will be needed to tag individual subsystems.

The generalization provided by the canonical equations lends strength to the argument that this is a valid attack on quantifying the quality of an energy system. Also, the practical value of this capability is high enough to allow a trade-off for a more complete descriptive model, with its attendant complexity. In actual practice the question of identity of system or subsystem equipment arises so seldom, and is so easily answered, that very little hardship in system evaluation is likely to occur for this reason. If, however, a universal quantitative measure of permutative characteristics should ever be developed, the information content proposed above can be modified to include this additional factor. It is not likely that this will be done easily or soon, since deriving such a measure is the crux of the problem of information retrieval which is at present defying ready solution. By avoiding the necessity of using it in system effectiveness analysis, a streamlined technique becomes possible which makes evaluation physically, mathematically, and economically feasible.

3.4.3 Applications to Physical Systems - The solutions of Laplace's equations which are commonly employed in engineering system analysis include Ohm's law, Newton's laws of mechanics, thermal and hydraulic flow laws, Bernouilli's equations, Kirchoff's laws, etc., which are deterministic forms. The normal probability distribution expression,

$$\frac{dp}{dx} = \frac{1}{\sqrt{2\pi\sigma^2}\ \exp\left(\frac{x^2}{2\sigma^2}\right)} \tag{19}$$

is another solution of Laplace's equation.

In general, these equations are too restricted for system analysis. They can be expanded by raising the parameters of voltage, current, force, displacement, time, temperature, heat, etc.,

to the level of power, and multiplied by time to obtain energy, or work done. Usually the equations obtained in this way are not directly usable for systems integration. A better method appears to be the formulation of the system characteristics in energy terms at the start, followed by treatment of the energy flow by the Laplacean, in which the $\nabla^2 E$ term is the force or concentration which causes flow, the $\frac{\partial E}{\partial t}$ is the actual flow, and the $K(\phi)$ is the function of the constraining conditions which limit or control the flow. The simplicity of this form, and the fact that it leads to all the laws of network theory, makes it an extremely practical technique for representing physical system performance.

When the quantities are statistical, simple substitution of their information content in the proper places in the equation provides the conversion needed to handle them adequately.

3.5 Prediction Theory - The preferred model of system effectiveness must be amenable to extrapolation as a time function to a future period, with a tolerable error probability. The essence of prediction is stability of the time functions. Obviously, infinitely stable functions are mathematical abstractions. Practically usable time functions of performance are always subject to increasing uncertainty as they are projected farther into the future, because of limitations in quantity and quality of our knowledge of them.

3.5.1 Statistical Prediction - Statistical prediction theory, as developed by Norbert Wiener, depends on the stationarity of the stochastic qualities of the performance time function. Perpetual stationarity of a system situated in an uncontrolled environment is a fiction, but limited or quasi-stationarity can be defined if the characteristics of the environment can be bounded for a finite time. Then it becomes possible to construct time functions of system effectiveness suitable for limited prediction of reliability, stability, maintainability, variability, etc., at some time subsequent to the time of prediction. It is also possible to assign progressive values of confidence in the accuracies of the predictions for successive future time. This type of limited prediction is facilitated by the ergodic hypothesis which relates time and phase averages of stationary random processes. (However, the ergodicity itself must be stationary with time.) Other complications, due to the involved nature of stochastic mathematics are also to be reckoned with. A conclusion which can be drawn from a study of statistical prediction is that the predictability depends on those qualities of a random time series which are not random, i.e., its causally connected connectivity, such as stationarity, correlatability, limitations on its probability distribution and variances, etc., which are averaged to provide a nonrandom, essentially continuous time function. While statistical methods are unavoidable, in describing the physical world, because of human error tendencies and narrow bandwidth for information reception, it is desirable to utilize as much of the well-established causally based prediction as possible. Although the controversial question whether the universe is continuous or discrete is at the root of this distinction, nevertheless, for reasons of expediency, it is desirable to depend on the remarkably successful humanly generated logically connected continuous models of the physical world. Speculations as to their realism are inapplicable in these partially abstract models (which are unrealistic in other ways also).

3.5.2 Deterministic Prediction - The classical deterministic laws of energy system performance are essentially statistical trends. They have been verified by sufficient experimental and observational data to make the variances sufficiently small, and our confidence in their stability for useful extrapolation sufficiently high, to permit quantizing the residual errors completely out of them. The transition threshold between statistical and deterministic treatment cannot ordinarily be defined, as it depends greatly on the amount of information available and on the use to which it is to be put. For example, a gambler may tolerate a 25 percent variance in past performance record of a horse on which he plans to bet, while a design engineer can not accept even 1 percent variation in the stability of Ohm's law. In practice, even when the laws are acceptably stable, the variations in the rates of deterioration in the quality of the materials used to control the energy in the desired channels can defeat an exact analysis or prediction. Consequently, the system engineer usually finds himself with a foot in each camp - deterministic and probabilistic.

To eliminate the discomfort of his position, a valid means of reconciling the differences between these two fields must be developed. Fortuitously, such a homogenizing process has already been provided for special problem areas, i.e., the entropy measure of availability of thermal energy. This can be expressed either probabilistically, as $-\sum_1^n p_i \log p_i$, or deterministically, as $\int \frac{dQ}{T}$. The equivalence of the potential gradient expressed as $\frac{dQ}{T}$ to the average information value of the probabilities p_i of molecular collisions, $-\sum_1^n p_i \log p_i$, provides a mechanism for conversion from one to the other, which is based on the diffusion equation,

$$\frac{{}^2E}{k} = -\frac{\partial E}{\partial t} = -\frac{dQ}{T} = -\Sigma p \log p \qquad (20)$$

Similar approaches in statistical mechanics and information theory establish the validity of the identity between the Laplacian $\nabla^2 E$ for the "lumpiness" of energy, and the information value of the subjective probability of occurrence of an energy event (or flow), as formulated in the mind of the observer, (subject to qualification by the accuracy of the observation technique).

3.5.3 Informational Prediction - The prediction of the static and dynamic performance effectiveness of information handling systems can be facilitated if the conservation of information can be established as a constraint in a particular system. Although there is a lack of agreement on this point for general systems, in which the entropy average is declared to increase continually, nevertheless, in isolated systems, the accounting of entropy can be performed without appreciable error when the entropy takes the form of "bound" information, which is associated with energy processes (the only type in which there is interest). It can be safely postulated that all real information is of this type since the existence of abstract information can only be imagined. Even in the brain there is a nonzero minute amount of energy associated with each bit of information.

Information system effectiveness can then be predicted confidently by extrapolation of the laws of information flow, as time functions:

$$C(t) \leq W(t) \log \frac{P(t) + N(t)}{N(t)} . \tag{21}$$

The probability of error in the prediction will be related to the stability of the time functions of coding efficiency, noise levels, bandwidths, etc., which prevail in the given system. However, in practical systems, the energy levels must be normalized before these relations can be applied.

There is a lack of agreement among authorities on this point of information conservation except in the case of "bound" information (Brillouin), or "self-information" (Fano). This is the information associated with energy processes, the only type that is considered.

Prediction of system effectiveness by extrapolation of the laws of information flow, considered as time functions

$$R(t) = W(t) \log_2 \left[\frac{P(t) + N(t)}{N(t)} \right] \tag{22}$$

is thus theoretically possible. The probability of prediction error will be a function of the stability of the time functions, i.e., coding efficiency, noise levels, bandwidths, etc., which prevail in the given system. In general, the recency of information theory as a mature science has inhibited the development of canonical laws with the same scope and degree of potency as those of energy physics. Consequently, it appears expedient to borrow from energetics, axioms and algorithms to explain and clarify information processes, by defining $\nabla^2 E$ as information content in Laplace's (Poisson's, Helmholtz's) equations. Thus both quality and quantity of performance effectiveness can be quantified at the same time in the same units of information measure, and their dynamic variations can be calculated and predicted within the limits imposed by the accuracy of initial system performance data.

3.6 SPAN Model

3.6.1 Philosophy - In accordance with the preceding analysis, Nortronics' philosophy of SPAN is presented here.

"The need for an exact, universal performance measure, suitable for expressing the dynamic laws of system behavior, is generally recognized. For several years, Nortronics Division of Northrop Corporation, at its Systems Support Department in Anaheim, California, has been actively engaged in research to define such a unit for use in systems engineering and design projects. It has been obvious that a merely statistical or semistatistical measure is inadequate for effective system integration and for determination of dynamic tradeoffs, because of the lack of causal coherence in a statistical representation. Logical causality is the principal basis of engineering design. A universal, mathematically descriptive "yardstick," and a modeling technique, which will take advantage of well known and understood logically dependent relationships and engineering principles, are eminently desirable.

"Such a measure is currently to be found in various limited areas of scientific technology. Examples are the entropy measure of availability of heat energy in thermodynamics, the Laplacian equation for the flow rate of energy density in classical physics, and the subjective measure of availability of decisions which is called information content flowor "negentropy" rate, in information and communication theory."

3.6.2 Problem - The resemblance between these quantities is not sufficient justification for using them together without a more mathematically rigorous validation of their correspondence. Another problem is the lack of a rigorous formal treatment of the dynamics of energy availability, as a criterion of system performance quality, in contemporary literature in the field of operations research.

Consequently, the study efforts supported by Nortronics - Systems Support analysts have been directed toward the solution of these difficulties. Their object has been to unify the variety of system performance measures, and to establish the dynamic behavior of this common denominator of performance, in operation, for effective system integration. The result has been designated the SPAN (System Performance Analysis) method. It requires additional validation to merit general adoption as a rigorous analytical tool. Nevertheless, in the absence of a comparably powerful method, it appears expedient to develop the SPAN approach on an experimental basis, while continuing to strengthen its theoretically formal foundation.

3.6.3 Solution

3.6.3.1 Common Unit of Measure - The universal performance unit chosen for SPAN (on an empirical basis, subject to more rigorous validation by further study) is the practical one of the product of a unit energy event and its availability factor (measured or estimated), and is called the pau (performance availability or effectiveness unit).

The prevailing dichotomy in systems analysis caused by the existence of objective measures of performance (such as those derived from the Laplacian of energy flow in mathematical physics) and of subjective measures (such as probability, information content and rate, and likelihood estimates), has created a dilemma which can only be resolved by an arbitrary definition of the relation between man and the physical world. The hypothesis adopted for SPAN postulates that the majority of physical phenomena of engineering interest can be treated as essentially deterministic, following the law of causality compatible with engineering principles. The effect of human coupling to the physical world, either by observation or measurement techniques or by control of physical processes, is comparable to that of a feedback loop in a servo system. However, there is an element of error in the human's appraisal of the significance of his own observations, and in the suitability of his control operations to produce a desired effect. Consequently, much of the human effort is devoted to the task of adaptively correcting for his imperfect coupling to the physical world. From a subjective viewpoint, the normalized objective energy availability product in observations or measurements can be described as information density, while the effective control of physical processes by engineering design and operation can be called performance. The unanalyzed imperfections or errors in observation are placed in the category of

"noise," while the result of residual imperfections in prediction or realization of desired performance in future time becomes unreliability. Using the SPAN method with these inefficiency factors inserted, a performance availability model of the relation between any human or group of humans, and any limited physical situation, natural or artificial (such as a weapons system), can be constructed, as illustrated in figure 1. It is also theoretically possible to minimize the physical elements and represent mainly human organizations, either lumped or distributed, in terms of these performance factors.

3.6.3.2 Functional Model - A major difficulty, that of placing the model on an exact functional basis rather than leaving it as an undefined intuitive concept, has been advantageously solved, in part, by recognizing the validity of the Laplacian equation for describing energy flow. This expression, in effect, represents the "lumpiness"[1] (the divergence of the gradient of potential) of energy in a distributed state. It can be equated to the rate of flow of energy, in the common forms employed by engineers, by simple formulas, one of which reduces to Ohm's law for simple electrical circuits. It can be shown that other laws of electrical circuits are also applicable to describing objective performance factor dynamics, if equivalent constraints exist between the actual system and its electrical analog.

As might be expected, the laws of information flow and the thermodynamic entropy exhibit the same formalism and can be rationalized in the same terms as the circuit laws since they are also based on energy. The SPAN method promises to make it possible to unify the objective and subjective fields of description of performance and to analyze systems which include both types, in quantified integrated techniques.

3.6.4 Formalism

3.6.4.1 Assumptions

(a) All systems are energized.
(b) All systems are causally connected (functionally interdependent).
(c) All systems are directly measurable.
(d) Probability is a measure of subjective degree of partial certainty.
(e) Energy potential theory is applicable to analysis of inelastic systems as follows:

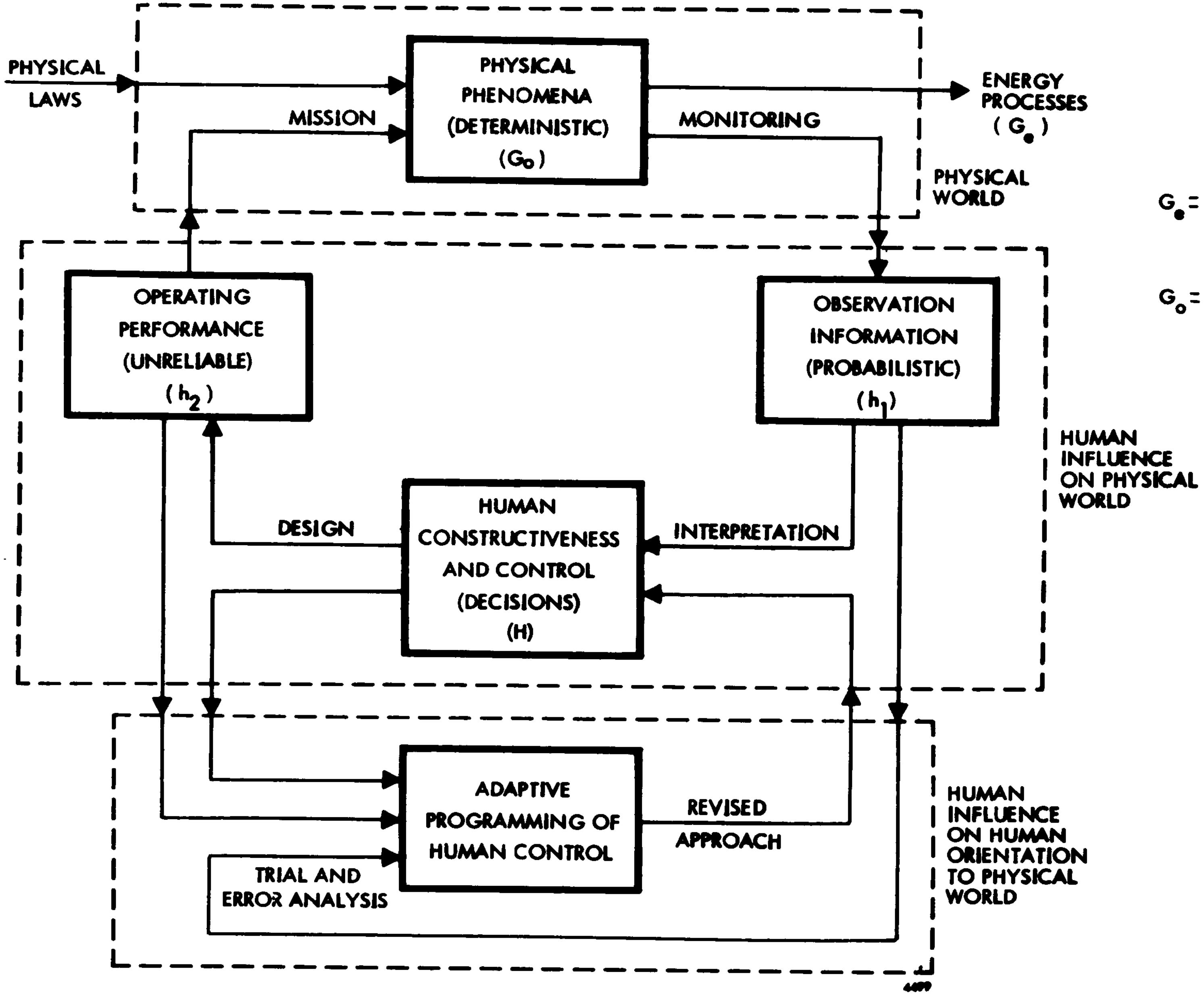
For determinate systems the Laplacian represents energy availability on a linear scale.

$$\frac{\nabla^2 E}{K} = -\frac{\partial E}{\partial t} = \bar{e} A \qquad (23)$$

For indeterminate systems the $\log_2$ of probability (entropy, negentropy) represents energy availability.

(1) Morse and Feshback, Methods of Theoretical Physics, pp 8, 51.

Figure 1. RELATION BETWEEN HUMAN AND PHYSICAL WORLD



$$G_e = \frac{G_o}{1 + G_o h_1 h_2 H}$$

$$G_o = \frac{G_e}{1 - G_e h_1 h_2 H}$$

$$H = -\Sigma\, p \log p = S \tag{24}$$

In mixed systems, therefore, an equivalence can be established between determinate and indeterminate measures of availability.

$$H(E) = -\Sigma\, p \log p = -\frac{\nabla^2 E}{K} = \frac{\partial E}{\partial t} = -\bar{e}\, A \tag{25}$$

(f) The pattern or structure complexity A as an availability function of energy (corresponding to the content of information) can also be quantified in terms of the logarithm of the number of decisions or choices of energy flow events. This usually can be obtained from the mathematical description, as, for example:

Number of dimensions
Number of degrees of freedom
Degree of exponent in power series of approximation
Order of highest harmonic in Fourier series approximation
Logical steps
Number of polynomial factors in transfer function
Average information content $(-\Sigma\, p \log p)$ of structure of conditional probabilities.

(g) System performance effectiveness ξ can be quantified as the difference of the product of the energy flow rate or availability vector $\bar{e}$ with the pattern complexity figure, A, and the information content $H(E)$.

$$\xi = \bar{e}\,(A - H(E))$$

(h) Limited resolution solutions are possible.

(i) Elastic systems, such as field-coupled types, can be analyzed using higher order equations.

$$\frac{\nabla^4 E}{K} = \frac{\partial^3 E}{\partial t^3} \tag{26}$$

3.6.4.2 Method - The quantification and organization of engineering system performance availability is accomplished by reconciling objective and subjective measures of energy processes. The universal nature of energy allows integrated synthesis and analysis to be made by the familiar techniques of potential theory. Defining probability as a subjective variable of uncertainty permits equating information measures of performance availability to the determinate measures of energy potential gradients. In potential theory the dynamic behavior of energy is expressed by Laplace's or Poisson's equation:

$$\frac{d\nabla^2 E}{dt} = -\frac{Kd}{dt}\left(\frac{\partial E}{\partial t}\right) \tag{27}$$

where K is a stable function of constraints. This can also be written

$$-\frac{d}{dt}\left(\frac{\partial E}{\partial t}\right) = \frac{d\nabla^2 E}{K\,dt} = \bar{e}\dot{A} \quad . \tag{28}$$

In information theory

$$H_{av} = -\Sigma\, p \log p \quad . \tag{29}$$

This agrees with the accepted definition of entropy (which has been rigorously proved to be identical with information content):

$$S = H = -k\,\Sigma\, p \log p \quad , \tag{30}$$

for the average energy quality or availability of a complex stochastically described energy process. The channel capacity formula

$$\frac{dH}{dt} = C \leq W \times \left[\log_2 (P + N) - \log_2 N\right] \tag{31}$$

suggests that information in a physical situation may be defined as a quality of energy and that it flows with energy. Thus, by allying energy and information, the gap between the abstract concept of dynamic information and the concrete idea of performance availability variation can be bridged. (This hypothesis is supported by recently published works of Professor E. T. Jaynes, developing the majority of the fundamental laws of statistical mechanics and of thermodynamics by statistical inference. His development results in the same measures of physical performance as those of information theory and is obtained in the same way.) Consequently, it now appears justifiable to postulate that subjective statistical performance data, and objective or classical physical potential theory measurements, can be brought together and evaluated by the same yardstick. Either static or dynamic performance may be treated in this manner.

Consequently, it appears that system integration engineers can safely utilize the hypothesis that every energy process is causally connected, and that the only sources of uncertainty are the states of ignorance of the human being involved in the system.

3.6.4.3 Applications - This cross-fertilization technique promises to make it easier to reduce large quantities of statistical data to simple expressions of physical laws (subject to the unavoidable errors in human observation). It also allows the possibility of discovery and exploration of new physical laws by combining statistical pattern recognition devices with potential analyzers. Another possible application is the use of coding theory for interpretation of physical and biological phenomena, from experimental measurements.

3.6.4.4 Noise - One result of this approach is an unconventional definition of noise. The most commonly used description of noise is that of an independent objective energy function, definable only by statistical measures. By way of contrast, the proposed SPAN method treats this same noise process in subjective terms, i.e., as a complex of incompletely analyzed or misunderstood coherent energy processes, some of which occur in the observation measurement ranges and in interpretation operations and are commonly called "errors." (Thus incomprehensible scientific theories can qualify as "noise" to certain levels of mentality, under conditions of inability to classify them more precisely and thoroughly.) In effect, the phenomena which we can recognize and explain fall within a certain scope of analyzability and are considered as significant, while other poorly understood phenomena which fall outside this band are expediently called "noise" or "error."

From this viewpoint, random function theory is not the most useful discipline for describing and analyzing noise processes since the concept of randomness or lack of causal coherence is opposed to that of intelligently organized information. Although the controversial negative slope of the time rate of change of universal entropy supports the hypothesis of a physically realizable random process, which would constitute an unsaturable energy "sink" (a "hole" in the fabric of the cosmos, so to speak), this rate, if real, is too slow to affect the majority of contemporary engineering designs. This rate often can be neglected in the system planning.

3.6.4.5 Coding - Another result of this method is equivalence between code optimization problems and those of impedance matching in electric circuits. Many of the perplexing aspects of coding theory become intuitively clear in the light of this analogy. For example, the simplest analog of an information channel is shown in figure 2.

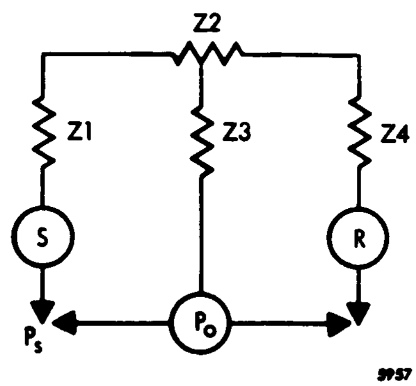


Figure 2. SIMPLE ANALOG CIRCUIT

In this circuit,

$$\frac{dq}{dt} = I_R \leq \left[\frac{1}{(Z_1 + Z_4)}\right] \times (E_S - E_O) \quad . \tag{32}$$

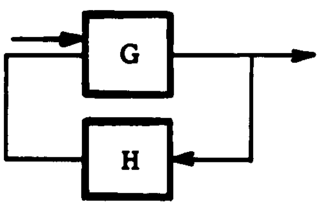
Here Z_1 represents the encoder, Z_2 and Z_3 the transfer and loss characteristics of the transmission medium, and Z_4 the decoder. E_S is the signal power.

Compare this with Shannon's channel capacity law:

$$\frac{dH}{dt} = C \le W \left[\log (P + N) - \log N\right] \quad . \tag{33}$$

It is obvious that a more exact expression is possible for equation (32) only when Z_2 and Z_3 of the transmission medium are specified with respect to E_S. Similarly, the optimum code for a particular information system described in equation (33) will be a function of the configuration of its noise sources and power losses. The product of I and E representing power in figure 1 is the analog of the product of performance energy by its availability and the information content of its probability factors. If this parameter is called performance power, it can be used to represent a more meaningful index of system performance than performance energy, availability due to its pattern complexity, and information content, can do individually. Then all the powerful methods of electric circuit analysis, such as impedance matching, can be applied to the system. Feedback circuits can be quantitatively designed; games situations can be simulated by infinite gain closed loops; and adaptive control criteria or error functions can be put into familiar circuit engineering form.

Also, vice versa, it is possible to apply the principles of circuit design directly to systems consisting of information channels exclusively. For instance, the effect of information feedback on coding efficiency can be deduced by considering, as in the following circuit, the closed-to-open-loop conversion equations; which are entirely valid if a normalized energy factor is included.



$$G_{eff} = \frac{G}{(1 + GH)} \tag{34}$$

$$G = \frac{G_{eff}}{\left(1 - G_{eff}H\right)} \tag{35}$$

Also, the concept of information power amplification can be introduced. It consists of increasing, by any means, the product of information rate and significance (code rank).

3.6.4.6 Modeling - The tremendous simplification obtained by this unification of subjective and objective performance descriptions pragmatically justifies an apostasy from the dogmatic principles of random theory. The elimination of the indigestible random process from the analytical treatment of system performance makes it possible to describe all the performance parameters (including the physically measurable contributions of the human elements) in terms of a common denominator, the performance availability or effectiveness unit (pau for short). Furthermore, the correspondence between the dynamic behavior of this unit and that of an electric charge, which is validated by Laplace's and Poisson's equations for energy flow, permits any lumped constant

system to be readily and accurately represented by an electrical model. This model, in the form of a network analyzer or simulator, is very amenable to manual or automatic variation. It can be quickly optimized for any selected criterion, such as minimum performance degradation, maximum stability, or maximum security against countermeasures. Maximum economy or value, as a pau function of relative cost, can be obtained if acceptably accurate functional relations between pau and cost are available. The effect of competitive supply and demand forcing functions in an open market situation will logically appear to cause limited, approximately linear, relations between cost and pau. When this can be determined to be true to the desired accuracy, actual reciprocal dollar costs can be substituted for pau and the model problem can then be optimized directly in terms of maximum economy or minimum waste.

3.6.5 Practicality - The urgency of the present requirement for a formalized methodology of system performance analysis seems to justify taking these apparent liberties with accepted theories. The lack of agreement among contemporary proponents of prevailing probability theories provides even stronger reasons for attempting to cut at one stroke the Gordian knot which for many years has defied attempts to untie it, i.e., the knot into which probability theory has become convoluted. Because of it the progress of system engineering techniques has been impeded and even stopped in some areas. As a result, many practical operating systems suffer from lack of integration, incompatibility, and poor proportioning. The pressure for more intelligent and effective system analysis will not permit waiting an indefinitely long time for a perfectly validated, rigorously proven, theoretical, universal basis for system integration. If the designers of the hammer had waited for the perfect nail-driving tool (some version of a nail-shooting gun), many of us would be living in sod huts or wigwams. The hammer is a poor tool because it transfers all its energy on impact, tending to bend or crumple the nail. It has insufficient guidance, and requires transforming rotational energy into rectilinear energy. It is inflexible and seldom matched to a particular nail or to the material receiving the nail. In spite of these disadvantages, the hammer has been the preferred tool for driving nails for centuries, because of sheer timeliness, availability, and economy.

The proposed SPAN methodology is offered on the same basis of expediency. It is an empirical tool, with a certain degree of basically sound design principles, and with some crudities. Experience with its use and more thorough study of its theoretical basis will undoubtedly provide some refinement. Meanwhile, its advantages for engineering system analysis promise to greatly outweigh its disadvantages.

3.6.5.1 Quantification - The significance of various parameters of system performance as partial projections of the overall effectiveness function can be recognized easily in the SPAN model. This category includes such variables as reliability, communication, data processing, compatibility, versatility, standardization, fail-safe operation, man-machine interface matching, and bandwidth. When these variables are represented by their dimensions as performance availability functions their reconciliation in a universal measure is obviously possible. They can be tentatively quantified and their effects calculated in a given system, using the system behavior principles given above. These objective system energy flow laws (developed from the Laplacian), and the corresponding dynamic subjective laws for the rates of charge of probability densities and conditional

dependencies, exemplified by Shannon's information channel capacity law, may perhaps eventually be shown to be special cases of some rigorous general law for energy-information distribution. Meanwhile their isomorphism is quite adequate for limited scope mathematical modeling of system effectiveness on a pragmatic basis.

3.6.5.2 Prediction - Prediction of system reactions, for adaptive programming of corrective action, may be greatly facilitated by SPAN by expressing system performance in pau, effectiveness numbers, using the electrical analog of current flow to simulate the flow of pau. The accuracy of prediction will be limited of course by the accuracy of the input data and of the analogs themselves. It also appears possible to evolve previously unknown system concepts and relations by derivation from the analogous electrical circuit model, drawing on the wealth of techniques in analysis of circuit and network engineering. Topological patterns may then be devised to explore the effects of changes in system configuration, either by planned strategy or as caused by casualties during operation.

3.6.5.3 Human Factors - Application of SPAN to systems including strong elements of human factors and organizations may also be validly justified. The degree of accuracy with which human performance availability can be measured will affect the accuracy of the final results. Fortunately, both the fields of psychology and of communications have provided extensive data on human performance which can be converted readily into pau.

When the organizational pattern of human systems is indefinite, analogs with corresponding qualities can be used. Such familiar techniques as simulation of distributed parameters, by resistive planes or volumes, with exploring probes, as employed in solution of problems in potential distribution, can be applied to this purpose. The unique versatility of the SPAN measure and its compatibility with the logically consistent flow laws, exemplified by electrical circuits, offers to provide more rigorous significant analysis of human factors than is ordinarily provided by exclusively statistical treatments. It is often true that available causally connected or functional information on human performance is neglected in statistical analyses because of lack of suitable techniques for including it. The SPAN method proposes to avoid this difficulty by integrating both coherent and incoherent information in the same treatment.

3.6.5.4 Programming - The SPAN technique, as thoroughly validated, can thus integrate the multitude of system philosophies and reconcile the varying shades of emphasis on interlocking parameters in an exact mathematical model defining explicitly all the energetic interactions between subsystems and inputs. "Grey" areas of indecision can be eliminated, and recognition of system coordination conditions can be greatly facilitated. It also offers to provide a means of coordinating and extending the methods of programming and data processing, as exemplified by the PERT technique. SPAN will furnish greater significance and more coherent functional analysis and prediction capabilities in complex system evaluation and scheduling than is possible with PERT alone.

3.6.6 Validity

3.6.6.1 Equivalence of Scalars - The validity of the equivalence of performance quality (entropy) and information content has been well established by A. I. Khinchin[2] and L. Brillouin[3] for distributed processes, such as the thermal state of a gas. Extending this equivalence to include the discretely lumped "black boxes" or functional blocks of a system can be performed by considering the "lumps" as aggregates of statistical state variables, for which the averages are sufficiently stable to be reducible to a deterministic quantity, with a tolerance on its stability. In practice, this tolerance may be neglected only if it is less than the probability of error arising from other causes. Nevertheless, it always exists. It therefore qualifies every "deterministic" quantity as actually statistical. It may seem queer to write Ohm's law as:

$$I = \frac{E}{R} \pm 1 \times 10^{-x} \quad , \tag{36}$$

but this is a more correct expression than the unqualified form:

$$I = \frac{E}{R} \, . \tag{37}$$

Thus, the probabilistic character of all human quantitative measures can be established, and the applicability of information/entropy measures verified. Hence,

$$p \left(I \,\middle|\, \frac{E}{R} \right) < 1 \quad , \tag{38}$$

etc.; and similarly for other measures, such as $P = I E$, for example,

$$p \left(P \middle| I_o, \; E_o \right) = p \left(I_o \middle| I_M \right) p \left(E_o \middle| E_M \right) \quad , \tag{39}$$

where M indicates a measured quantity. Practically, if

$$1 - p(x) \rightarrow o \quad , \tag{40}$$

it is possible to normalize the negligible uncertainties in the measured quantities in a system by assuming that the tolerances or variances are all equal (but not zero!). Then the actual scale factors of energy levels become the critical values, and their logarithms (base two) provide the information values, which are called availability or effectiveness numbers, with respect to any arbitrarily selected reference level used in the model.

3.6.6.2 Vectors - When vector quantities or other orthogonally ordered set of quantities are involved, a decision must be made whether or not to retain their dimensional identifications. If

(2) Khinchin, A. I., Mathematical Foundations of Information Theory, Dover Publications, Inc., 1957.

(3) Brillouin, L., Science and Information Theory, Academic Press, 1956.

retained, a higher order mathematical model, such as a ring, field, or vector space type will be required. The nondimensional nature of probability, and of its information measure ($\log_2 p$), makes such higher order models unnecessary for purely mathematical reasons since p acts as a universal solvent of the combinational as well as of the singular quantities in question. However, for convenience, it may be desirable to construct individual models representing one- or two-dimensional projections of an n-dimensional system model. This can be done, for instance, by retaining the laws of combination of the projections, such as the 4-D law:

$$r^2 = x^2 + y^2 + z^2 - c^2t^2 \quad . \tag{41}$$

3.6.7 Utility - Specific problem areas in weapons systems operations, which may be advantageously analyzed by the proposed SPAN method, include the following.

3.6.7.1 Mission Effectiveness - The evaluation of mission effectiveness as a function of performance availability of an entire system, can be expressed by SPAN techniques as a product of the energy state or pattern of the weapon, the probability of availability of its performance in the prescribed pattern, and the compatibility of the performance pattern with the desired mission. These factors can be measured either in exact engineering units or in subjective appraisals of probability such as reliability or information content.

3.6.7.2 Reliability and Maintainability - The optimum trade-off between built-in reliability versus corrective maintenance can be calculated from the open-loop equivalent of the closed-loop system including the maintenance functional feedback, in terms of time and cost.

3.6.7.3 Versatility - Both the mission function and associated support and maintenance functions can be expressed as variables. The performance and cost economics of such flexibility can be determined as results of an adaptive control system design optimizing exercise of the electrical analog model.

3.6.7.4 Automation - The degree of automation can also be optimized for a particular design by adaptive control theory. In fact, it is one of the variables considered in the analysis of versatility, described above.

3.6.7.5 Methods of Repair - Methods of repair, including throwaway parts, module repair, redundant circuit switching, and piece-part repair, can be compared in terms of their effect on overall system performance and on the logistic bottlenecks created.

3.6.7.6 Standardization - The levels of standardization, as a compromise between logistic supply cost and maximized performance by periodic redesign, can be optimized easily by the SPAN method of modeling.

3.6.7.7 Failure Diagnosis - Techniques of failure diagnosis are amenable to exact analysis, as representations of the information circuit configurations or codes, in the mathematical model. Relative efficacy can be deduced from the analogous impedance matching effectiveness in the circuit.

3.6.7.8 Repair Function Adequacy - Repair function adequacy can be evaluated in the same manner as that used for failure diagnosis.

3.6.7.9 Duty Cycle - The optimum duty cycle and allowable down-time of both the weapon and the maintenance and support equipment can be calculated for any given readiness time requirement, expressed either as an average or as an absolute limit.

3.6.7.10 Man/Machine Functions - The special man/machine problem of safety can be studied more analytically than contemporary intuitive methods allow. The definition of safety in terms of maximum pau's coupled to the human system provides a common denominator for both mission performance and safety, and allows estimation of tradeoffs, both intentional and accidental.

3.6.7.11 Cost - The criterion of cost, given a fixed or inflexible budget, is easily applied to force a maximum value solution. This occurs since the correlation function between pau's and dollars appears to be simply definable, with acceptable accuracy, in typical systems.

3.6.7.12 Command and Control Channel Optimization - Command and control channels can be optimized by SPAN, in relation to the integration of complete systems, since the identity between information units and availability units can be utilized. A unified solution of combined systems is consequently possible.

3.6.7.13 Monitoring Display and Readout - The design of monitoring display and readout techniques, as part of the human factors engineering program, can be reviewed with considerable insight into problems ordinarily treated by psychological and information theory approaches, without extensive system functional integration.

3.6.7.14 Computer Use - The part played by the ubiquitous computer in system performance, maintenance, and control can be defined in more specific performance units than is now possible using conventional computer functional descriptions. Compatibility between the computer and other subsystems, and analysis of special problems peculiar to computers, such as error-correcting codes, can be integrated by SPAN into the general system performance optimizing problem. The decision as to how much computing to use in a particular system can then be made on a firm factual basis.

3.6.7.15 Allowable Error Tolerances - Allowable error tolerances can be assigned on a logically sound and mathematically exact basis. Unnecessarily broad tolerances, which reflect the safety factor for ignorance of true error accumulation effects, can be avoided by SPAN analysis.

3.6.7.16 Universal Type of Test - The possibility of a universal type of test, suitable for any engineering measurement in any typical system configuration, becomes plausible. The universal nature of the SPAN energy availability "yardstick" giving an effective value of negentropy, enables a universal transducer and a universal complexity evaluator to be visualized. These basic measures are universal because they ignore the specialized pattern uniqueness that distinguishes one humanised design from another. This does not appear to be an insuperable handicap in engineering

testing, because seldom is the system configuration unknown to the test operator. It therefore appears superfluous to require the test equipment also to include specialized system pattern recognition. In almost every test situation, a numerical figure of merit for any subsystem, representing the energy level and availability, with a number for the complexity, will suffice for checking readiness. It is almost an axiom that, by the law of entropy increase, any deterioration in performance will cause a decrease in the figure of merit assigned. Hence, no failure will remain undetected by a numerical test. An exception would be the effect of human experimentation or sabotage which might, by a remote possibility, alter the subsystem function to an equivalent noncompatible pattern, without degradation of the figure of merit. However, this exception will ordinarily be negligible. Consequently, it appears that a universal performance tester, capable of checking any engineering device, is entirely within the bounds of reasonable expectation, with a limited amount of research and development effort.

3.6.7.17 Strategic Planning and Programming - The coherent dynamic behavior model of military planning and operating strategy, which the SPAN approach provides, will make possible more effective integration of military missions, objectives, implementation, support technology and socio-economic environments than is available by conventional operations research. Its extension to include games theory situations can be readily accomplished.

3.7 SPAN Language - The intricacies of translating a system concept successively into a SPAN mathematical model, a SPAN physical model or analog, and an actual system design can be minimized by the use of the following rules of translation, lists of transforms, and conversion factors. To cover the wide variety of physical and engineering performance functions completely, an encyclopedic treatment is needed. The lists given here, however, do not include all possible categories nor do they cover all possible parametric performance variables. The more familiar and significant variables and functions of design engineering, found in communications, control, power, and instrumentation systems, have been listed but no claim is made for completeness.

Table 1 compares the dimensional units of the dynamic and the energetic systems with the SPAN units. The SPAN units are shown in two incompletely normalized forms incorporating the signatures of time and length, and in the absolute forms comprising only energy level and information (entropy, availability).

Table 2 shows the transformations from the relations of static energy domain states to their SPAN counterparts, both in the mathematical and physical models. Table 3 gives the transform rules for the dynamic or time rate versions of those laws. Table 4 shows the spatial rate or gradient transforms, and table 5 indicates the developments of the parametric forms and laws by successive differentiation. Table 6 shows the relationships between probability, availability, information, reliability, stability, and effectiveness.

Table 1. DIMENSIONAL UNITS AND CONVERSIONS

Quantity	Dynamic System	Energetical System	SPAN Spatial	Temporal	Absolute
Length	L	L	L	$T^{1/2}$	$E^{1/2}A^{-1/2}$
Time	T	T	L^2	T	EA^{-1}
Velocity	LT^{-1}	LT^{-1}	$E^{-1}AL$	$E^{-1}AL$	$E^{-1/2}A^{1/2}$
Mass	M	$EL^{-2}T^2$	AL^4	AT^2	E^2A^{-1}
Force	MLT^{-2}	EL^{-1}	AL	$AT^{1/2}$	$E^{1/2}A^{1/2}$
Pressure	$ML^{-1}T^{-2}$	EL^{-3}	AL^{-1}	$AT^{-1/2}$	$E^{-1/2}A^{3/2}$
Momentum	MLT^{-1}	$EL^{-1}T$	EL	$ET^{1/2}$	$E^{3/2}A^{-1/2}$
Energy	ML^2T^{-2}	E	E	E	EA^0
Power	ML^2T^{-3}	ET^{-1}	EL^{-2}	ET^{-1}	E^0A
Torque	ML^2T^{-2}	E	E	E	EA^0
Temperature	t	t	EA^{-1}	EA^{-1}	EA^{-1}
Heat	ML^2T^{-2}	E	E	E	EA^0
Thermal Capacity	$L^2T^{-2}t^{-1}$	$L^2T^{-2}t^{-1}$	$E^{-3}A^3L^2$	$E^{-3}A^3T$	$E^{-2}A^2$
Thermal Conductivity	$MLT^{-3}t^{-1}$	$ELT^{-1}t^{-1}$	AL^3	$AT^{3/2}$	$E^{3/2}A^{1/2}$
Emissivity	$MT^{-3}t^{-1}$	$ET^{-1}t^{-1}$	AL^{-2}	AT^{-1}	$E^{-1}A^2$
Entropy	$ML^2T^{-2}t^{-1}$	Et^{-1}	A	A	E^0A
Electric Charge	Q	EV^{-1}	A	A	E^0A
Displacement	QL^{-2}	$EV^{-1}L^{-2}$	AL^{-2}	AT^{-1}	$E^{-1}A^2$
Electric Field Intensity	$MQ^{-1}LT^{-2}$	VL^{-1}	L	$T^{1/2}$	$E^{1/2}A^{-1/2}$
Capacitance	$M^{-1}Q^2L^{-2}T^2$	EV^{-2}	AL^{-2}	AT^{-1}	$E^{-1}A^2$
Current	QT^{-1}	$EV^{-1}T^{-1}$	AL^{-2}	AT^{-1}	$E^{-1}A^2$
Voltage	$MQ^{-1}L^2T^{-2}$	V	L^2	T	EA^{-1}
Resistance	$MQ^{-2}L^2T^{-1}$	$E^{-1}V^2T$	$E^{-1}L^6$	$E^{-1}T^3$	E^2A^{-3}
Magnetic Flux	$MQ^{-1}L^2T^{-1}$	VT	L^4	T^2	E^2A^{-2}
Induction	$MQ^{-1}T^{-1}$	$VL^{-2}T$	L^2	T	$E^{-1}A$

M = mass, L = length, T = time, t = temperature, Q = electric charge, E = energy, V = voltage, A = availability.

Table 2. FUNCTIONAL TRANSFORMS FOR STATIC ENERGY - AVAILABILITY MODELS

Math Model	Transform Law	Physical Model in Analog Parameters ($\overline{\ }$)
MECHANICAL $E = FL = ML^2T^2$ $= fs = mas$ $= mgh$ $= \frac{mv^2}{2} = \frac{m}{2}\left(\frac{ds}{dT}\right)^2$ $= ms\frac{d^2s}{dT^2} = ms\frac{dv}{dT}$ $= mv\frac{dv}{ds}$ $= \frac{I}{2}\left(\frac{d\theta}{dT}\right)^2 = \frac{I\omega^2}{2}$ $= m\tau\theta = mr^2\alpha\theta$ $= \frac{1}{2}mr^2\omega^2$ s = L = distance v = velocity a = acceleration	$\bar{e}A = \frac{\nabla^2 E}{K} = -\frac{\partial E}{\partial T}$ A = availability E = energy	$A = \overline{qV} = \overline{CV^2}$ (electrical) $A = \frac{\overline{W}}{\overline{m}} = \overline{FL}$ (mechanical) $A = \overline{S} = \overline{Qt}$ (thermal) $A = \frac{\overline{W}}{m} = \frac{\overline{pv}}{m}$ (thermdynamic)
ELECTRICAL $E = PT = VIT$ $= \frac{V^2T}{Z} = I^2ZT$ $= \frac{CV^2}{2} = \frac{LI^2}{2}$ V = voltage I = current	$\bar{e}A = \frac{\nabla^2 E}{K} = -\frac{\partial E}{\partial T}$	

Table 2. FUNCTIONAL TRANSFORMS FOR STATIC ENERGY - AVAILABILITY MODELS (Continued)

Math Model	Transform Law	Physical Model in Analog Parameters ($\frown$)
THERMAL $Q = tS$ $pv = Rt$	$\bar{e}A = \frac{\nabla^2 E}{K} = -\frac{\partial E}{\partial T}$	
E. M. RADIATION $s = \vec{V} \times \vec{H}$ $= \frac{\vec{V} \cdot \vec{D} + \vec{B} \cdot \vec{H}}{2\sqrt{\epsilon\mu}}$	$\bar{e}A = \frac{\nabla^2 s}{K} = -\frac{\partial s}{\partial T}$	
ACOUSTICS $E = \frac{\rho v^2}{2} + \frac{p^2 c^2}{2\rho}$	$\bar{e}A = \frac{\nabla^2 \phi}{K} = -\frac{\partial \phi}{\partial T}$ ϕ = acoustic energy	

Table 3. FUNCTIONAL TRANSFORMS FOR DYNAMIC ENERGY - AVAILABILITY MODELS

Math Model	Transform Law	Physical Model in Analog Parameters ($\frown$)
MECHANICAL $\frac{dE}{dT} = \frac{LdF}{dT} + \frac{FdL}{dT}$ $= m\frac{Ld^3L}{dT^3}$ $\doteq m\frac{da}{dT} + \frac{adm}{dT}$	$\bar{e}\frac{dA_T}{dT} = \frac{d}{dT}\left(\frac{\nabla^2 E}{K}\right)$ $= \frac{d}{dT}\left(-\frac{\partial E}{\partial T}\right)$	$\frac{dA_T}{dT} = \frac{\overbrace{Vdq}}{dT} = \overbrace{iV} = \frac{\overbrace{V^2}}{Z}$ (electrical)

Table 3. FUNCTIONAL TRANSFORMS FOR DYNAMIC ENERGY - AVAILABILITY MODELS (Continued)

Math Model	Transform Law	Physical Model in Analog Parameters ($\bar{\ }$)
ELECTRICAL $\frac{dE}{dT} = P$ $= T\left(\frac{IdV}{dT} + \frac{VdI}{dT}\right)$ $+ VI$ $= \frac{V^2}{Z} + \frac{Td}{dT}\left(\frac{V^2}{Z}\right)$ $= I^2Z$ $+ T\left(\frac{I^2dZ}{dT} + \frac{2IZdI}{dT}\right)$	$\bar{e}\frac{dA_T}{dT} = \frac{d}{dT}\left(\frac{\nabla^2 E}{K}\right)$ $= \frac{d}{dT}\left(-\frac{\partial E}{\partial T}\right)$	$\frac{dA_T}{dT} = \frac{d\overline{W}}{\overline{m}\,dT}$ $= \frac{1}{\overline{m}}\left(\frac{\overline{F}d\overline{L}}{dT} + \frac{\overline{L}d\overline{F}}{dT}\right)$ $= \frac{\overline{L}d\overline{a}}{dT}$ etc. (mechanical) $\frac{dA_T}{dT} = \frac{d\overline{S}}{dT} = \frac{\overline{Q}d\overline{t}}{dT} + \frac{\overline{t}d\overline{Q}}{dT}$ (thermal)
THERMAL $\frac{dQ}{dT} = \frac{tdS}{dT} + \frac{Sdt}{dT}$ $\frac{dE}{dT} = \frac{pdv}{dT} + \frac{vdp}{dT}$	$\bar{e}\frac{dA_T}{dT} = \frac{d}{dT}\left(\frac{\nabla^2 E}{K}\right)$ $= -\frac{d}{dT}\left(\frac{\partial E}{\partial T}\right)$ $= \frac{dS}{dT} = \frac{Q}{t}$	$\frac{dA_T}{dT} = \frac{d\overline{W}}{\overline{m}dT}$ $= \frac{\overline{p}d\overline{v}}{\overline{m}dT} + \frac{\overline{v}d\overline{p}}{\overline{m}dT}$ (thermodynamic)
E. M. RADIATION $\frac{d}{dT}(\vec{V} \times \vec{H}) = \frac{dS}{dT}$	$\bar{e}\frac{dA_T}{dT} = \frac{d}{dT} = \left(\frac{\nabla^2 E}{K}\right)$ $= -\frac{d}{dT}\left(\frac{\partial S}{\partial T}\right)$	

Table 3. FUNCTIONAL TRANSFORMS FOR DYNAMIC ENERGY - AVAILABILITY MODELS (Continued)

Math Model	Transform Law	Physical Model in Analog Parameters (⌐)
ACOUSTIC $\frac{dE}{dT} = \rho va + \frac{c^2 pdp}{\rho dT}$	$\bar{e} \; \frac{dA_T}{dT} = \frac{d}{dT}\left(\frac{\nabla^2\phi}{K}\right)$ $= \frac{d}{dT}\left(\frac{-\partial\phi}{\partial T}\right)$	

Table 4. FUNCTIONAL TRANSFORMS FOR SPATIAL GRADIENT ENERGY - AVAILABILITY MODELS

Math Model	Transform Law	Physical Model in Analog Parameters (⌐)
MECHANICAL $\frac{dE}{dL} = L\frac{dF}{dL} + F$ $= mL\frac{d}{dL}\left(\frac{d^2L}{dT^2}\right)$ etc.	$\bar{e} \; \frac{dA_L}{dL} = \frac{d}{dL}\left(\frac{\nabla^2 E}{K}\right)$ $\simeq \frac{\nabla^3 E}{K}$ $= \frac{d}{dL}\left(\frac{-\partial E}{\partial T}\right)$	$\frac{dA}{dL} = \frac{\bar{V}d\vec{\bar{q}}}{dL} = \bar{V}\,\mathrm{grad}\,\vec{\bar{V}}$ (electrical) $\frac{dA}{dL} = \frac{d\bar{W}}{\bar{m}\,d\bar{L}}$ $= \frac{1}{\bar{m}}\left(\bar{F} + \bar{L}\frac{d\bar{F}}{d\bar{L}}\right)$ $= \bar{L}\frac{d\bar{a}}{d\bar{L}}$ (mechanical)
ELECTRICAL $\frac{dE}{dL} = \frac{d}{dL}\left(\frac{CV^2}{2}\right)$	$\bar{e} \; \frac{dA_L}{dL} = \frac{d}{dL}\,\frac{\nabla^2 E}{K}$ $= -\frac{d}{dL}\,\frac{\partial E}{\partial T}$	

Table 4. FUNCTIONAL TRANSFORMS FOR SPATIAL GRADIENT ENERGY - AVAILABILITY MODELS (Continued)

Math Model	Transform Law	Physical Model in Analog Parameters ($\overline{}$)
THERMAL $\frac{dQ}{dL} = \frac{tdS}{dL} + \frac{Sdt}{dL}$	$\bar{e}\,\frac{dA_L}{dL} = \frac{d}{dL}\left(\frac{\nabla^2 E}{K}\right)$ $= -\frac{d}{dL}\left(\frac{\partial E}{\partial T}\right)$	$\frac{dA}{dL} = \frac{d\overline{S}}{dL} = \frac{\overline{Q}\,d\overline{t}}{dL} + \frac{\overline{t}\,d\overline{Q}}{dL}$ (thermal) $\frac{dA}{dL} = \frac{d\overline{W}}{mdL}$ $= \frac{\overline{p}\,d\overline{v}}{\overline{m}dL} + \frac{\overline{v}\,d\overline{p}}{\overline{m}dL}$ (thermodynamic)
E. M. RADIATION $\frac{d}{dL}(V \times H) = \frac{dS}{dL}$	$\bar{e}\,\frac{dA_L}{dL} = \frac{d}{dL}\left(\frac{\nabla^2 S}{K}\right)$ $= \frac{d}{dL}\left(\frac{-\partial S}{\partial L}\right)$	
ACOUSTIC $\frac{dE}{dL} = \frac{d}{dL}\,\frac{\rho v^2}{2}$ $+ \frac{d}{dL}\,\frac{p^2 c^2}{2\rho}$	$\bar{e}\,\frac{dA_L}{dL} = \frac{d}{dL}\left(\frac{\nabla^2 \phi}{K}\right)$ $= \frac{d}{dL}\left(\frac{-d\phi}{dT}\right)$	

Table 5. PARAMETER CONVERSION BY SUCCESSIVE DIFFERENTIATION

Physical	Other	Name	Energy Derivation
MECHANICAL			
L	$s,\ h$	Length	$E = L\,\frac{d^2L}{dT^2} = as = gh$
$\frac{dL}{dT}$	$v,\ \dot{s}$	Velocity	$E = \frac{1}{2}\,m\left(\frac{dL}{dT}\right)^2 = \frac{1}{2}\,mv^2$
$\frac{d^2L}{dT^2}$	$a,\ \ddot{s},\ \dot{v}$	Acceleration	$E = mL\,\frac{d^2L}{dT^2} = mas = Fs = W$
$\frac{d^3L}{dT^3}$	$a,\ \dddot{s},\ \ddot{v}$	Jerk, surge, acceleration rate	$E = \frac{m}{2} \int L\,\frac{d^3L}{dT}\,dT$
$\frac{d}{dL}\left(\frac{d^2L}{dT^2}\right)$	$\frac{da}{ds}$	Acceleration gradient	$E = \frac{m}{2} \int L\,\frac{d^2L}{dT^2}\,dL$
$\frac{d^2}{dL^2}\left(\frac{d^2L}{dT^2}\right)$	$\frac{d^2a}{ds^2} \cdot \nabla^2 a$	Div. grad a Laplacian "lumpiness" availability	$E = \frac{m}{2} \iint L\,\frac{d^2L}{dT^2}\,dL\,dL$ $= \frac{m}{2} \int \nabla^2 a\,dT$
$\frac{d}{dT}\left[\frac{d^2}{dL^2}\left(\frac{d^2L}{dT^2}\right)\right]$	$\frac{d}{dT}\left(\frac{d^2a}{ds^2}\right)$	Availability flow	$\frac{dE}{dT} = \frac{d}{dT}\left(\frac{d^2a}{dL^2}\right)$
ELECTRICAL			
Q	$-\nabla^2 V$	Charge	$E = \frac{QV}{2}$
$\frac{dQ}{dT}$	$I,\ \dot{Q}$	Current	$E = \int IVdT$
$\frac{dQ}{dL}$	V	Voltage	$E = \int IVdL = \frac{TdQ}{dT}\,\frac{dQ}{dL}$

Table 5. PARAMETER CONVERSION BY SUCCESSIVE DIFFERENTIATION (Continued)

Physical	Other	Name	Energy Derivation
ELECTRICAL (Continued)			
$\frac{d^2Q}{dT^2}$	$\dot{I}$, $\ddot{Q}$	Radiation	$E = k\ddot{Q} + C$
$\frac{dQ}{dT}\ \frac{dQ}{dL}$	P	Power	$E = PT$
$\frac{d^2Q}{dL^2}$	$\frac{dV}{dL}$	Voltage gradient (drop)	$E = \frac{V^2}{Z}$
$\frac{d^3Q}{dL^3}$	$\nabla^2 V = - K \frac{\partial V}{\partial T}$	Charge concentration Laplacian "lumpiness"	$E = \int \nabla^2 V dT$
$\frac{d^4Q}{dL^4}$	$\nabla^2 E$	Energy concentration	$E = \int\int \nabla^2 E\, dL\, dL$

Table 6. DIMENSIONAL TRANSFORMS PROBABILISTIC

Function	Transform Law (Mathematical Model)	Physical Model in Analog Parameters ($\frown$)
PROBABILILITY		
$(0 \le p \le 1)$	$H = - \log p_i$	$\overset{\frown}{qV}$
$p(FL\|E) = p_E$	$H_{av} = - \sum_{i=1}^{n} p_i \log p_i$	$\sum_{i=1}^{n} \overset{\frown}{q_i V}$
$p(mas\|E) = p_E$		
$p(Vg\|E) = p_E$		
$p(PT\|E) = p_E$		
$p(pv\|E) = p_E$ etc.		

Table 6. DIMENSIONAL TRANSFORMS PROBABILISTIC (Continued)

Function	Transform Law (Mathematical Model)	Physical Model in Analog Parameters ($\frown$)
AVAILABILITY $(1 \leq A \leq \infty)$ k = combinatorial scale factor (nonordered coincidence) K = permutation scale factor, such as $K!$ (ordered coincidence)	$A = \log k$ $A_t = \Sigma\ k \log k$ $A = K$ $A_t = \Sigma\ K$	$\overset{\frown}{qV}$ $\Sigma\ \overset{\frown}{qV}$
INFORMATION $H = S$ (entropy)	$H = -\log p_i$ $H_{av} = -\sum_{i=1}^{n} p_i \log p_i$	$\overset{\frown}{qV}$ $\Sigma\ \overset{\frown}{qV}$
EFFECTIVENESS $\ell = f(\eta, \log H, A, S)$	$\xi = -\log H + A + p_c$	$\overset{\frown}{qV}$ $\Sigma\ \overset{\frown}{qV}$
RELIABILITY $R = E(T) = \exp\left(-\frac{T}{m}\right)$ $= p(E \mid T)$	$\xi(T) = -\log H(T) + A(T)$ $= -c\frac{T}{m}$	$\overset{\frown}{qV}(T)$
STABILITY $S = f(R)\ \alpha\ \frac{dR}{dT}$	$\frac{d\xi}{dT}(T) = -\frac{d \log H}{dT} + \frac{d}{dT} A(T)$	$\frac{d\overset{\frown}{qV}}{dT} = \overset{\frown}{iV}$

3.7.1 Definition of Availability - It is desirable to define availability exactly to minimize sematic confusion. For this purpose the following examples will be helpful:

(a) If 1 white ball and 9 black balls are mixed, the probability of drawing (in random manner) the white ball is 0.1. The information value is $-\log_2 (0.1) = 3.2$ bits, which is a measure of the uncertainty which is resolved when the white ball is drawn.

(b) If the black balls are removed leaving only the white ball, the probability of selecting it is unity, and the information value is zero.

(c) If then, 9 white balls are added to the original white ball, making 10 white balls, it is plausible to assume that the probability of selecting a white ball could be greater than one. However, the definition of probability precludes this, as unity probability is absolute certainty which cannot be exceeded. Nevertheless, it is obvious that the addition of the nine white balls might have added something to the situation by virtue of their combinational nature (if any).

A more suitable illustration of this effect is that of an electric charge, for instance, 1 microcoulomb in a capacitor. The probability of receiving a 1 microcoulomb charge by connecting a zero resistance lead to the condenser terminals is unity. If 10 microcoulombs are stored in the condenser, the same probability of receiving 1 microcoulomb charge with a zero resistance connection exists. (Of course, the other 9 microcoulombs also would be received.) However, if the lead resistance were not zero, the first 1 microcoulomb discharge would be received much more rapidly if there were 10 than if there had been only one. Obviously, the presence of the additional charges made the first discharge, more available, because of the interrelationship of mutual repulsion between the charged electrons, in view of the constraint of the resistance in the connecting lead. Hence the availability value does not stop at unity, as did the probability value, but increases above that value according to the force which causes the energy event to have a given time rate of occurrence. Thus, availability depends on the environment as well as the initial situation. By the use of a logarithmic scale for the representation of availability, the information value of the probability of occurrence contained in the original situation, and the availability representing the effects of the forcing and constraining or impeding functions, can be combined compatibly in a common term.

For instance, if it were possible to identify a certain group of electrons in the total charge, a probability of receiving that group in the first microcoulomb discharge could be computed as $p(q)$. The effectiveness of that particular group at the receiving point would then be

$$\xi = A - H = -\log_2 p(q_i) + \log_2 \sum_{i=1}^{n} \frac{q_i}{CR} \quad . \tag{42}$$

If the energy field has a vector character, so that its effectiveness is a function of direction, then the availability will also be a vector, in its simplest form. If the directionality is considered as a ratio of two values (i.e., the directional derivative), then in effect the yardstick is changed from

a point measure to a line measure. This incompatibility can be removed by a number of expedients, such as:

(a) The vectors can be resolved into their projections or components in each dimension, the logarithms of each dimensional measure added together. This is jusitifiable if the dimensions are orthogonal, and truly represent independent degrees of freedom.

(b) If the field function is reducible to algebraic form, then the individual terms can be treated for probability and availability, and their average can be obtained by the $H_{av} = -\Sigma\ p \log p$ formula for average information content.

(c) If neither of these techniques is applicable, separate models of the tractable regions can be constructed, and connected together at the real time-space level.

3.7.2 Significant Parameters - The significant parameters into which the conventional performance descriptions are converted, consist of energy and availability and information content. It is possible to retain the character of length, time, mass, or electric charge if desired, as shown in table 1. However, while this may be done without prejudice to the mathematical model, it introduces a limitation to the physical or analog model which represents the mathematical model. Consequently, it should be used with care when necessary, and the data should be converted back into the absolute form whenever possible.

The form $A - \log H$ of the performance figure of merit can be refined further to resemble the abstract information form H if the scale factor of the energy pattern is included in A, which effectively normalizes it with respect to energy. To prevent confusion, this combined parameter should be distinct from availability A and information H. It will be called "Effectiveness" or code rank figure of merit, with the symbol ξ. In this form it can be used interchangeably with $\log H$ in information computations, and with

$$A = \frac{\nabla^2 E}{K} \tag{43}$$

in energy availability computations. Expressions containing both forms can then be integrated and interrelated in a single joint function of ξ, and quantified in performance availability units (pau).

3.7.3 Examples - The following examples illustrate the technique of deriving ξ, the figure of merit, from conventional expressions of performance:

Example 1. Product of factors $(x_1\ x_2\ x_3\ \ldots)$ If p_j is simply conditioned

$$\xi = A - H = + \log \prod_{\substack{i=1 \\ j=1}}^{n} x_i p_j = \sum_{i=1}^{n} \log x_i + \sum_{j=1}^{n} \log p_j \ . \tag{44}$$

Otherwise

$$\xi = \sum_{i=1}^{n} \log x_i + \sum_{j=1}^{n} p_j \log p_j \quad . \tag{45}$$

Example 2. Sum of variables $(x_1 + x_2 + x_3 + \ldots)$

$$\xi = A - H = A - H(\Sigma x_i) = x_1 \log x_1 + x_2 \log x_2 + x_3 \log x_3 + \ldots \tag{46}$$

Example 3. Combinations of sums and products $(ax^2 + bx + c)$

$$\xi = A - H = ax^2 \log ax^2 + bx \log bx + c \log c \quad . \tag{47}$$

Example 4. Transcendental function $(\sin x)$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots \tag{48}$$

$$\xi = A - H = x \log x + \frac{x^3}{6} (3 \log x - \log 6) + \frac{x^5}{120} (5 \log x - \log 120) \; . \tag{49}$$

Example 5. Transfer function (energy, not voltage). Since energy $= Pt = \frac{V^2 t}{Z}$, the familiar voltage form of the conventional transfer function $v(t)$ must be converted to $Pt(t)$ or $\frac{V^2(t)\,t}{Z}$.

3.8 Simulation

3.8.1 Electrical Network - As implied above, simulation of system performance by an electrical network analog, using SPAN, is entirely practical. The facility of experimentally varying both the forcing functions of required performance of a mission, under selectable conditions of environment, and the system parameters, organization pattern, and efficiency for a given mission, makes dynamic system analysis feasible. The rapid response of electrical analogs provides quick readout and display of the performance and of the interrelations and tradeoff possibilities. The simulation may then be used as a real-time monitor of system operating condition, or as an optimizer of mission strategy.

3.8.1.1 Tradeoffs - Tradeoffs or crossover points in the relative emphasis on competing performance factors can be recognized and expressed either in the units of performance availability or in equivalent cost-dollars. In a competitive production and operation economy, the continuous effect of supply and demand equilibrium-forcing often results in a quasi-linear or simple functional relation between dollars and pau. This may hold over a useful portion of the procurement range with a reasonable degree of accuracy. Thus, SPAN will allow the tradeoffs to be made in terms of budget dollars. The overall mission also can be priced as a pau per dollar value, and can be

optimized manually, or automatically on an analog computer. Digital computing techniques may be substituted for analog methods (with some loss of flexibility), wherever existing facilities provide greater economy by utilizing available digital computing equipment.

3.8.1.2 Analogs - The conventional electrical network analyzer, consisting of series and shunt impedances, power sources and sinks, is an excellent vehicle for SPAN system simulation. As shown in figure 3, the typical system functions of performance energy generation and organization, and of energy transfer and attenuation, are represented by their respective analogs of electric charge sources and constraints. The availability and/or information content of a normalized energy event at any point will be represented by the analog voltage, while the flow or rate of occurrence of these events will become the analog current. The impeding effect of constraints in the flow channels leading up to the measured point will be the ratio of the analog voltage and current. As in the analog, the impedance parameter may be calculated independently from information on its structure, or derived indirectly by division of the voltage by the current to give its transfer characteristics. Constant power, rather than constant voltage or current, supplies are used to simulate the performance inputs. The output of the simulation is also in terms of power, representing performance effectiveness flow.

The concept of reactive analog components is still somewhat nebulous, since it involves the idea of phase. System performance phase has not been defined with sufficient rigor at this time to allow specification of an analog simulation technique.

3.8.1.3 Reliability - Reliability functions of performance can be readily simulated as impedances with a programmed decay rate (in impedance) corresponding to the mean time to failure law.

3.8.1.4 Maintenance - Maintenance, in its simplest aspect, can be simulated by the process of restoring a degraded impedance to its original level, either as a continuous process or intermittently. When the information leading to the decision to perform maintenance is obtained from a monitoring or test and checkout operation, maintenance becomes a feedback function and conventional closed-loop feedback circuits can be used. Steady-state or sampled-data analyses are applicable as appropriate. The logistics of supply of active energy, of stored entropy (in the form of manufactured parts), and of personnel, can be represented by an energy flow channel with typical impedance values, providing electrical power to the maintenance loop gain element.

3.8.1.5 Maintainability - Maintainability then can be analyzed as the derivative, with respect to time or space, of the maintenance function. An operational amplifier connected as an integrator will furnish an index of maintainability as a time function. A space gradient can be derived as the quotient of two incremental spatial measurements. The full meaning of performance effectiveness is contained in the product of the energy and its availability-information value. In the analog this corresponds to the product of voltage and charge. The time rate or flow of performance effectiveness is then the result of multiplying voltage and current to obtain power, which is its analog.

3.8.1.6 Optimization - The simulation network can be optimized by maximizing the output for any desired criterion, such as total performance, maximum reliability (availability divided by

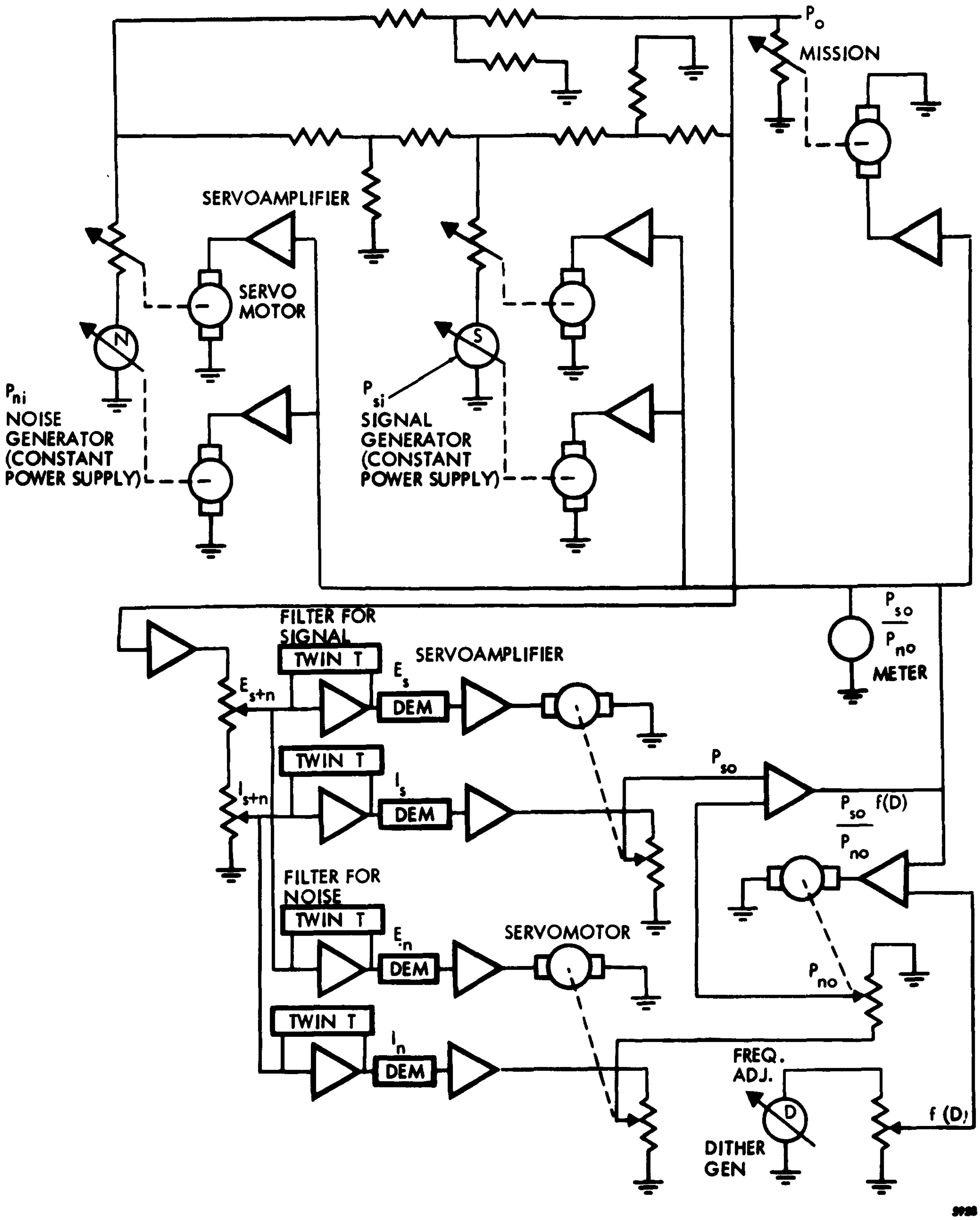


Figure 3. ELECTRICAL NETWORK FOR SYSTEM SIMULATION

time),minimum error (maximum information, i.e., availability), minimum transmission losses, etc.

3.8.1.7 Redundancy - Redundacy in the form of excess energy availability can be analyzed in terms of the analogous parameter of excess voltage in a component. It is obvious that a reliability "leak" or rate of degradation will allow an accelerated decay of performance with more complex systems, such as those containing redundancy with its attendant "switch-in" circuits. Hence, there obviously will be a point of diminishing return beyond which additional redundancy will cost more, in performance effectiveness per dollar, than the expense of a spare system. This is readily demonstrated by the network simulator.

3.8.1.8 Environment - Environmental effects can be inserted in analog form, as active or passive elements (such as current generators and shunt impedances) in the network.

3.8.1.9 Communication - Communication and data processing systems are modeled in the same manner, using the information content as part of the effectiveness number. Its association with energy level eliminates the incompatibility which ordinarily exists between information systems and energy systems. The degree of complexity of coding and decoding corresponds to the impedance or effectiveness concept of energy systems, and is represented by the same parameter.

3.8.1.10 Human Factors - Representation of human performance can be made on an empirical basis also. Although the human system has many degrees of freedom in internal programming and motivation, still there is strong evidence of a fixed limited capacity to transfer or originate information/energy. Further work in this area is necessary to put available psychological data in the proper form for validation of a "standard man."

3.8.1.11 Economic Value - Economic value as an index of performance effectiveness per dollar can be maximized by applying weighting factors of cost to the performance parameters of the system. This procedure is valid to the degree of accuracy to which the cost functions of performance are known.

It is obvious that the model constructed in this manner can be analyzed and optimized with a digital computer if the circuit equations are written. Optimizing methods include those of calculus of variations, Lagrange multipliers, successive approximation, and Monte Carlo sampling. Effectiveness for various mission "loads" can be easily determined in this way.

3.8.2 Mechanical Simulation - Mechanical models of system effectiveness dynamics can be utilized when the ruggedness of mechanical elements is needed for accuracy and reliability. Although it is difficult to provide the degree of flexibility of the electrical network, still, in some applications such as system monitors, the mechanical or electromechanical analog may be preferable to the pure electrical form. Mechanical linkages, gear ratios, clutches, frictional loads, etc., can be satisfactorily substituted for their analogous electrical counterparts. This technique will be useful in untended systems, where the attrition of severe environments prohibit less durable equipment.

3.8.3 Thermal Network - Thermal models, although not in general use, may be made to represent system performance. The common employment of "entropy" as a measure of thermodynamic and thermal efficiency is a "ready made" introduction to SPAN effectiveness philosophy and terminology, as the concept of energy-availability flow is easy to visualize in terms of heat-temperature flow. The Laplacian $\nabla^2 E$ then represents the "hot spots" in a thermal continuum. This type of model is especially valuable for simulating distributed systems or subsystems. Conventional thermal generators, insulators, flow channels, attenuators and amplifiers (heat pumps) may be used advantageously.

3.8.4 Distributed Simulation - The modeling of distributed energy-information fields and dynamics is somewhat restricted by the state-of-the-art of simulation. Circumscribed modeling is possible with such arrangements as electrolytic tanks, resistive lines and sheets, and thermal or optical continua. Although these techniques are generally more objectionable because of the difficulty of isolating and controlling the models, they lend themselves more expeditiously to two- and three-dimensional models. In combination with lumped system models they make it possible to simulate almost any phenomenological state or condition found in man-made and natural systems.

4.0 CONCLUSIONS

The conclusions drawn from this study, subject to additional validation, are as follows:

(a) System effectiveness can be quantified in a universal measure, comprising energy and effectiveness.

(b) Information systems and physical engineering systems can be analyzed in terms of the same parameters, by postulating that effectiveness $\xi = A - H$ where A is availability and H is information content.

(c) The dynamic performance of system effectiveness can be analyzed according to the following principles:

(1) The Laplace-Poisson-Helmholtz equations for energy availability:

$$\nabla^2 E = KE(t) \quad (50)$$

if

$$K = f(A, H) \, , \quad (51)$$

$$A = \log K \quad , \quad (52)$$

$$H_{av} = -\Sigma p \log p \quad . \quad (53)$$

Where

K = scale factor with respect to arbitrary reference level
p = probability
A = availability
H = average information content.

(2) A modified information channel capacity formula:

$$\bar{e}C = f(\rho_c) \; W \log_2 \frac{E_s + E_N}{E_N} \quad , \tag{54}$$

where

C = information density/flow
$\bar{e}$ = unit energy
$f(\rho_c)$ = coding efficiency
W = range divided by resolution (resolving power) of channel
E_s = desired (signal) energy
E_N = undesired (noise) energy

(d) Compatible system integration of information and physical energy systems can be accomplished by the union of the two factors or measures, E and ξ.

(e) Mathematical models for prediction of system effectiveness can be formulated using these principles.

(f) Physical models or simulators of system effectiveness also can be constructed and exercised to provide analog solutions of system problems.

(g) The system performance analysis (SPAN) methodology is capable of applying the above-mentioned universal system effectiveness measure and dynamic flow laws to the problem of prediction of future performance and of providing simulation techniques for obtaining valid solutions.

(h) Extension of the SPAN methodology to include elastic systems appears feasible.

(i) The following subordinate problems can be adequately solved by this method:

Reliability
Maintainability
Transmission effectiveness
Error minimization
Redundancy efficiency
Environmental effects
Man-machine interface
Logistics of supply
Economics
Optimization

(j) System integration can be effectually realized by employment of SPAN techniques.

4.1 Suitability of the SPAN Models - The SPAN models, mathematical and physical, are inherently compatible with contemporary performance analysis and metrology. Their advantages include:

(a) Sound theoretical basis
(b) Universal applicability (including human factors)
(c) Coherent dynamic behavioral principles
(d) Simplified mathematics
(e) Practical simulation
(f) Extreme flexibility
(g) Ready translation into conventional engineering terminology.

Its disadvantages include:

(a) Instruction requirements
(b) Disagreement with "random" theory
(c) Limited applicability to elastic systems
(d) Lack of identification "signature" during analysis.

4.2 Alternative Approaches - Among the most forceful methods of system analysis, other than SPAN, are the following:

(a) H. Paynter - Energy System Analysis
(b) M. D. Mesarovic - Multivariable System Control Theory
(c) V. Pugachev - Random Variable System Theory
(d) L. S. Pontajagin - Optimal Control Processes
(e) J. G. Truxal - Automatic Control System Theory
(f) C. Shannon - Information Theory and Communication Theory
(g) Wald - Statistical Decision Theory

(h) R. Bellman - Dynamic Programming
(i) S. S. Stevens - Psychophysical Metrics Power Law
(j) J. von Neumann - Reliability/Redundancy
(k) Kron and LeCorbeiller - Tensor and Matrix Analyses of Networks
(l) E. T. Jaynes - Information Theory Applied to Thermodynamics and Statistical Mechanics

These techniques, while affording great analytical power in restricted problem areas, have not provided the universality necessary for complete system performance extrapolation and integration. The methods which approach this objective most closely are the energetic analysis of Paynter and the informational (entropic) thermodynamics of Jaynes.

Generically, all these methods suffer from insufficient perspective through over-concentration in particular areas of interest, which has created a mutual estrangement of philosophies and an incongruence of phraseologies that have inhibited the evolution of a unified system theory.

5.0 RECOMMENDATIONS

The following recommendations are based on the study and analysis presented in the foregoing sections.

5.1 Expansion of SPAN Scope - The methodology entitled "SPAN" should be studied and expanded in theoretical scope and in reduction to practice, to establish, and increase its value as a systems analytical tool. This should be done as described in the paragraphs that follow.

5.2 Further Validation of SPAN Theory - Further validation of SPAN theory should include:

(a) The theoretical basis of the consolidation of information theory and energetics should be more thoroughly validated and extended, to conform with existing science, and to furnish a more valuable scientific analysis methodology.

(b) Compatibility with the specialized methods, such as system analysis mentioned in paragraph 5.2, should be developed. Also, a broader common language and set of transforms should be provided to obtain the maximum benefit of a truly unified system theory and formalism.

(c) Techniques of application to specific system problem areas should be further explored and elaborated and their effectiveness increased.

On a practical basis an experimental approach to validation of SPAN's efficacy should be conducted to show the flexibility, speed, coherence, rigor, and practicality of this method. This can be accomplished by the following:

(a) SPAN-type analyses should be performed of complex existing systems, for which sufficient performance information is available, for comparison with the SPAN results and prediction.

(b) SPAN analysis should be applied to a stubborn design or operating problem which has previously resisted solution by other methods. A pronounced improvement in insight into system idiosyncrasies, or in progress toward a successful solution, would then be a necessary (but not necessarily sufficient) proof of the validity of the SPAN philosophy.

(c) The advantages of the real-time solution rate of a SPAN analyzer as a system status display function should be evaluated by incorporation into a practical display system, associated with an existing command and control network.

(d) The possibility of a more exact representation of human performance in mixed systems, afforded by the universal SPAN metric, should be investigated by coordination of existing human performance data with a SPAN model of a "standard man."

5.3 Practical Evaluation Program - If the expected improvement in system efficiency by more effective integration and optimization techniques provided by SPAN is to be realized, its application to the refinement of the operation of existing large scale command and control systems, for optimization of system design, should be expedited, so that costly prototype and preproduction destructive testing and operating failures can be minimized.

The following program of evaluation is suggested as suitable for demonstration of the utility of SPAN for prediction of reliability and for optimization of system behavior.

(a) A typical command and control system (or discrete subsystem), such as the 117-L, will be modeled on a SPAN simulator, using design data and information on operating environment. The results of the SPAN analysis will be compared with reliability and maintenance records to verify the predictability of performance of the system. A study of this magnitude will probably require 3 man-years of effort.

(b) An especially obstinate or refractory system problem (such as human information capacity in large scale display design, anticollision warning, data system maintainability, etc.) will be modeled on a SPAN simulator to disclose the fundamental reasons for the difficulty of solution. This project will presumably be accomplished with 1 to 2 man-years of effort.

(c) An extreme environment (ECM, natural noise, combat attrition, artificial interference or other critical system condition) will be SPAN-simulated in conjunction with a strategically important command and control type system to predict its probability of effective performance in selected modes of operation. The results will be contrasted with a Monte Carlo type sampling from an analysis made by other methods, to show the improved consistency, logical strength, and significance of the SPAN solutions.

5.3.1 Techniques - The techniques employed for modeling will be varied to demonstrate the adaptability of SPAN analysis to analog, digital, DDA, and special purpose computers (such as the hybrid type). Its utility as a localizing pilot analysis prior to complex programming of a problem for a large scale digital computer will be investigated. The suitability of various methods of displaying the system coherence characteristic provided by SPAN in a monitoring presentation also will be studied.

5.3.2 Suggested Schedule - Because of the timeliness of this methodology and the acute need for obtaining real-time system solutions, it is suggested that the preceding proposed projects be scheduled as soon as possible, and with as short a program as is practical. It is probable that evaluation program 5.3 (a) above can be completed in 6 months, and (b) and (c) can be accomplished in 4 months each. Simultaneous treatment of a single system will result in considerable economy in the use of computing facilities, but would cause some inefficiency in manpower utilization, so that all three tasks might be performed together in 8 months.

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GLOSSARY

algorithms	special process for solving a particular problem
analog	parameter having properties similar to those of another parameter
apostasy	defection or breach of faith
availability	tendency of an energy event to occur; as a definition, the logarithm (base two) of the energy scale factor
availability factor	factor proportional to availability
average effectiveness	effectiveness averaged over an interval
bandwidth	specifically, frequency range in cycles per second, generally, any measurement range in minimally resolvable increments
bit	binary digit
bound information	information associated with energy
canonical equations	form-preserving set of equations
causal coherence	continuity depending on cause-effect logic
causality functions	cause-effect logical functions
causally connected	sequentially logically connected
channel capacity	maximum information rate of a communication channel
code rank	degree of complexity of a code or constructive pattern; as a definition, number of degrees of freedom or orthogonal dimensions in a pattern or code

coherence	dependence
conservative	placing bounds on a variable, bounded
constructive complexity level	see code rank
deterministic	based essentially on a high degree of certainty
deterministic form	form without any element of probability
deterministic function	function of variables without probability factors
dichotomy	double choice
distributed constraints	constraints described as states
distributed processes	processes concerned with states
distributed systems	systems consisting of states
effectiveness	quality of energy events; as a definition, the sum of the availability, code rank, and information numbers
effectiveness function	quality function
effectiveness number	see effectiveness
efficacy	effectiveness
energetics	theory of energy processes
energy-bit	unit energy event
energy event	change of energy
energy metric	standardized fundamental measure of energy
energy state	dimensional character of energy
entropy	unavailability of energy
environment	energy states adjacent or contiguous to a given energy region

environmental conditions	particularized environments
ergodic	a system is ergodic if it tends in probability to a limiting form which is independent of the initial conditions
ergodic hypothesis	interchangeability of the time-space distributions of an ergodic stationary random process
figure of merit	number expressing quality
form factor	see code rank
gestalt	coherent characteristic
ground state load	infinite sink, with zero reaction
heuristic	by trial and error
hiatus	gap, interval
homomorphic	having a unique correspondence
incipient	beginning
index of maintainability	figure of merit of maintainability
information	logarithmic measure of uncertainty
information content	logarithmic measure of unresolved uncertainty
information density	temporal/spatial derivative of information
intuitionistic level	optimizing level
isomorphic	having a unique one-to-one correspondence
kernels	essential functions
logical causality	cause and effect logic
logical structure	logical proposition structure
lumpiness	the divergence of the gradient, Laplacian ∇^2

maintainability	figure of merit for maintenance functions
manifold	dimensional complex
meta-language	super or abstracted language
metric of performance	fundamental figure of merit
mission function	energy function associated with mission or load
motional impedance	constraining effect on motion
multidirectional functionals	vector spaces
negentropy	negative of entropy
network simulator	circuit simulating a network
orthogonally ordered sets	sets of independent quantities
parametric	variable representing another variable or combination of variables
pattern measure	see code rank
pattern number	see code rank
pattern rank	see code rank
pau	performance availability unit
perceptron	pattern recognition device
performance	as a definition, the time derivative of energy event functions
performance effectiveness	as a definition, the effectiveness of the time derivative (or flow) of energy event functions
performance energy	energy associated with performance
performance events	discrete changes of performance energy

performance quality	performance effectiveness
performance time function	variability of performance with time
PERT	Program Evaluation and Review Technique
pragmatic	empirical
probabilistic form	in terms of probabilities
probabilistic function	function of probabilities
probability	subjective estimate of uncertainty, normalized to make $p = 1$ for complete certainty
quantification	reduction to numerical form
redundancy	excess over minimum requirements
reliability	performance time function
reliability leak	loss of reliability
self-information	see bound information
simply conditioned	singly connected
SPAN	System Performance Analysis
state space	dimensional space describing a certain state
stochastic	random time characteristic
symmetrical matter tensor	tensor describing the general conservation of matter (exclusive of mass-energy conversion)
thermal model	model utilizing the flow of heat
topological patterns	flow graphs
tractable region	region of feasible solution techniques
unit energy event	change of a unit of energy

universal performance measure	**measure of performance applicable to any energized phenomenon**
universal quantitative measure	**universal numerical measure**
unreliability	**negative of reliability**
variance	**statistical norm of uncertainty**

APPENDIX I
TYPICAL PROBLEM TREATMENT BY SPAN SIMULATION

INTRODUCTION

The process of reducing the problem specification to a mathematical model, and subsequently to a physical analog model, for evaluation and optimization, will be explained in this section. This process involves a translation from conventional language into SPAN terminology, and vice versa when the solution is obtained.

For purposes of illustration, a system consisting of an AN/FPS-20 search radar, nonevasive target, human operator, radio data link, data processing computer, data display, human system administrator, and command link to the mission region is hypothetically represented. Tentative values have been assigned to portions of the system for which firm quantitative data is unavailable.

SYSTEM VALUES

The hypothetical system values are assigned as follows:

	Functions	Specifications	Effectiveness Number
			(1 bit = 3 db approximately) Reference level - 1 watt
(1)	Radar Transmitter:		
	Average power	4.32 kilowatts(2^{13}watts)	+13 bits
	Pulse width	2.5 microseconds	Convert to range resolution
	PRF	360 pulses/second (2^{8+})	+9 bits
	Dual channel	Redundancy switching	+1 bit
	Reliability	97 percent/channel	-0.044 bit
	Antenna efficiency	50 percent(2^{-1})	-1 bit
	Beamwidth	1.33° (3 db)	Reference level
	Azimuth sweep	360° (2^{8+} degrees)	+8 bits (approximately)

	Functions	Specifications	Effectiveness Number
	Sweep rate	5.0 rpm $\frac{30°/\text{sec}}{1.33°}$	4.5 bits/second
	Range resolution	3000 feet	Reference level
	Velocity resolution	10 feet/second	Reference level
	Range bandwidth	100,000 feet/3000 feet = 33 (2^5) each way	+5 bits each way = 10 bits
(2)	Propagation:		
	Attenuation	1.2 db/n mi (1 bit = 3 db)	+0.4 bit/n mi
	Tentative target distance	50 n mi x 1.2 db/n mi = 60 db each way	+20 bits each way
(3)	Target:		
	Effective width	3 yards (0.01 mils)	-12 bits
	Reflectivity loss	50 db (1 bit = 3 db)	-16.7 bits
	Velocity	320 feet/second	+5 bits
(4)	Receiver (Normal):		
	Antenna loss	-3 db	-1 bit
	Signal gain	85 db	+28 bits
	Bandwidth	200 kc (2^{17} cps)	+17 bits
	Output level	2 volts (600 ohms) 6 milliwatts	-8 bits
(5)	Manual Readout to Plotting Board:		
	Optical input level	10^{-16} ergs/second (10^{-23} watts) (-230 db)	-77 bits
	Human internal gain	250 db (to 75 watts)	+83 bits
	Human output	75 watts x 10/decisions/second	+9.5 watt-bits
	Average efficiency	10 percent (-10 db)	-3.3 bits
(6)	Automatic Scanner and Analog-Digital Converter Readout:		
	Output level	1 milliwatt (-30 dbw)	-10 bits
	Output rate	1000 cycles/second (2^{10}cps)	+10 bits

	Functions	Specifications	Effectiveness Number
(7)	Radio Frequency Data Link:		
	Transmission loss	42 db (1 bit = 3 db)	+14 bits
	Bandwidth (frequency)	2 kc (2^{11} cps)	+11 bits/second
(8)	Data Processing Computer: (with inputs from other radars, also)		
	Energy level	1 milliwatt (-30 dbw)	-10 bits
	Scale factor (binary digits)	1	0 bits
	Bandwidth	100 kc (2^{16} cps)	+17 bits
	Error probability	90 percent accuracy	-0.15 bit
	Gain (to restore level)	30 db	+10 bits
			+17 bits
(9)	Computer Readout and Transmission Link to Display:		
	Attenuation in energy level	40 db	-13 bits
	Word rank	5 binary digits	+5 bits
	Output rate (printer)	100 words/second	+6.6 bits
(10)	Display:		
	Energy level	10 watts ($2^{3.3}$watts)	+3.3 bits
	Dimensions	3	
	Resolution (position)	1 percent spatial in 2 dimensions	+6.6 bits
	(velocity)	2 percent time in 1 dimension	+5.6 bits
	Number of targets (maximum)	10 degrees of freedom	+10 bits
(11)	Visual Optics:		
	Dimensions	3	+3 bits
	Angular range	120°	+11.5 bits

	Functions	Specifications	Effectiveness Number
	Angular resolution	0.05°	Reference level
	Energy level	10^{-15} watts (-150 dbw)	-50 bits
	Velocity resolution	1 percent each dimension	+14 bits
(12)	System Administrator and Controller:		
	Bandwidth	10 bits/second	+10 bits/second
	Optical input threshold	10^{-16} ergs, (10^{-23} watts) (-230 dbw)	-77 bits
	Human internal gain	250 db	+83 bits
	Human output (vocal)	1 watt maximum	0 bits
	Command complexity (pattern number)	4 logical steps	4 bits
			20 total
(13)	Verbal Command Link:		
	Bandwidth	2500 cps (2^{11} cps)	+11 bits
	Energy level	1 watt	0
	Attenuation	30 db	+10 bits
	Pattern rank (of modulation)	9th harmonic resolution	+9 bits
(14)	Command Decisions of Mission:		
	Unit decisions per second	25 (typical)	+25 bits

PROBLEM AREAS

It is planned to prepare a SPAN model of this system and demonstrate the methodology and techniques of analyzing the following problem areas and system characteristics by simulating them in an electrical network analog.

(1) Reliability
(2) Maintainability
(3) Transmission phenomena

(4) Error rates
(5) Redundancy - Component level
(6) Redundancy - Function level
(7) Natural environments
(8) Artificial environments
(9) Man-machine compatibility
(10) Economic factors
(11) Logistics
(12) Optimization.

Methodology

The method of attack on the conversion of system parameters into SPAN effectiveness numbers is illustrated in the following table:

Physical Parameters	Transform Operation	Electrical Analog
Unit energy $\bar{e}$ (reference level)	$\log_2 \bar{e} = 0$	0
Scale factor K	$\log_2 K = A$	A (volt-coulombs)
Dimension (bandwidth = range/resolution)	$\sum_1^n \log_2 BW_i = \rho_c$	ρ_c (volt-coulombs)
Degrees of freedom n		
Uncertainty (probability) p	$-\log_2 p = H$	H (volt-coulombs)
Effectiveness = $\bar{e}K$ BW n p	$\xi = A + \rho_c - H$	ξ (volt-coulombs)
Performance = $\frac{d}{dt}$ (effectiveness)	$\frac{d\xi}{dt} = \frac{dA}{dt} + \frac{d}{dt}\rho_c - C$	$\frac{d\xi}{dt}$ (volt-amperes)

The type of mathematical model of the system is selected from the various ways in which the circuit equations can be represented, such as Kirchoff's voltage and current laws, matrix analysis, topological "trees," relay or information network equations (Boolean logic) etc. The physical network analog then can be made to represent any condition of the mathematical model by using the appropriate components. Suitable analogs consist of the following:

Effectiveness number (active) - Power source with proper power level
Effectiveness number (passive) - Resistor
Transfer channel - Resistive "T" network
Output performance - Output power.

The apparent simplicity of this simulation is deceptive. It does not represent a simple conversion from a mechanical to an electrical quantity, for example, but instead, a model of energy flow. Thus the conventional electrical circuit, when modeled by SPAN, will not be simulated by its exact counterpart, but will show, for instance, the loss of energy through conversion into heat as

a shunt resistive leg. Also, it will show the main path through an amplifier as that from the power supply to the load, rather than from the controlling input signal to the load. Other differences become apparent as experience is gained in its use.

Simulation

The block diagram in figure A-1 shows the paths of performance flow in the selected system. The electrical analog of the system is shown in figure A-2. The initial exercise, after the proper values have been assigned to the power supplies and the resistors, is to evaluate the sensitivity of the power output, at the "mission," to the variation of each parameter which is amenable to change. Then, by the method of Lagrange multipliers, the sensitivities are adjusted (varying the resistances individually) to obtain the maximum power output for a fixed total of resistance in these resistors (representing a fixed budget). Although the values shown do not reflect cost factors, these can be readily inserted by applying weighting factors of relative cost to each performance parameter analog. In the example shown, an arbitrary conversion factor of 2,000 ohms per bit was used for expediency.

The optimum values can be obtained experimentally by a successive approximation technique, or by using feedback servo adjusting circuits which achieve the same result. If all weighting factors and parameters are equally influential on the output, the maximum will be obtained when all the sensitivities are equal. This can occur for a large range of adjustments, so that the constraints of approximating the sensitivities of uncontrolled parameters, such as free-space radio attenuation, also can be satisfied with reasonable accuracy. Other constraints, such as fixed budgets, maximum ratios of desired-to-undesired signals, games, or competitive situations, readily can be met.

Sensitivity ($\Delta P_o / \Delta R_i$)

When the system shown was tested for sensitivity, the results given in figures A-3 to A-8 were obtained. The discrepancies between individual sensitivities indicate that large degrees of "unbalance" exist in the original concept of the system. A first approximation optimizing attempt to bring these parameters into better correspondence with the mean value of the resistances of all the elements produced a 160-percent improvement in the output power, shown on page A-20. This was realized without violating the economic constraint of a fixed total budget of the sum of the resistances in question, representing performance quality functions. This indicated that the corresponding system would be similarly improved by a redistribution of the quality of performance throughout the system.

Reliability

To illustrate the problems of reliability, etc., a portion of the system was modeled in greater detail, showing the maintenance and logistics loops. This system, in block diagram form, is shown in figure A-9, and its SPAN electrical analog simulation in figure A-10.

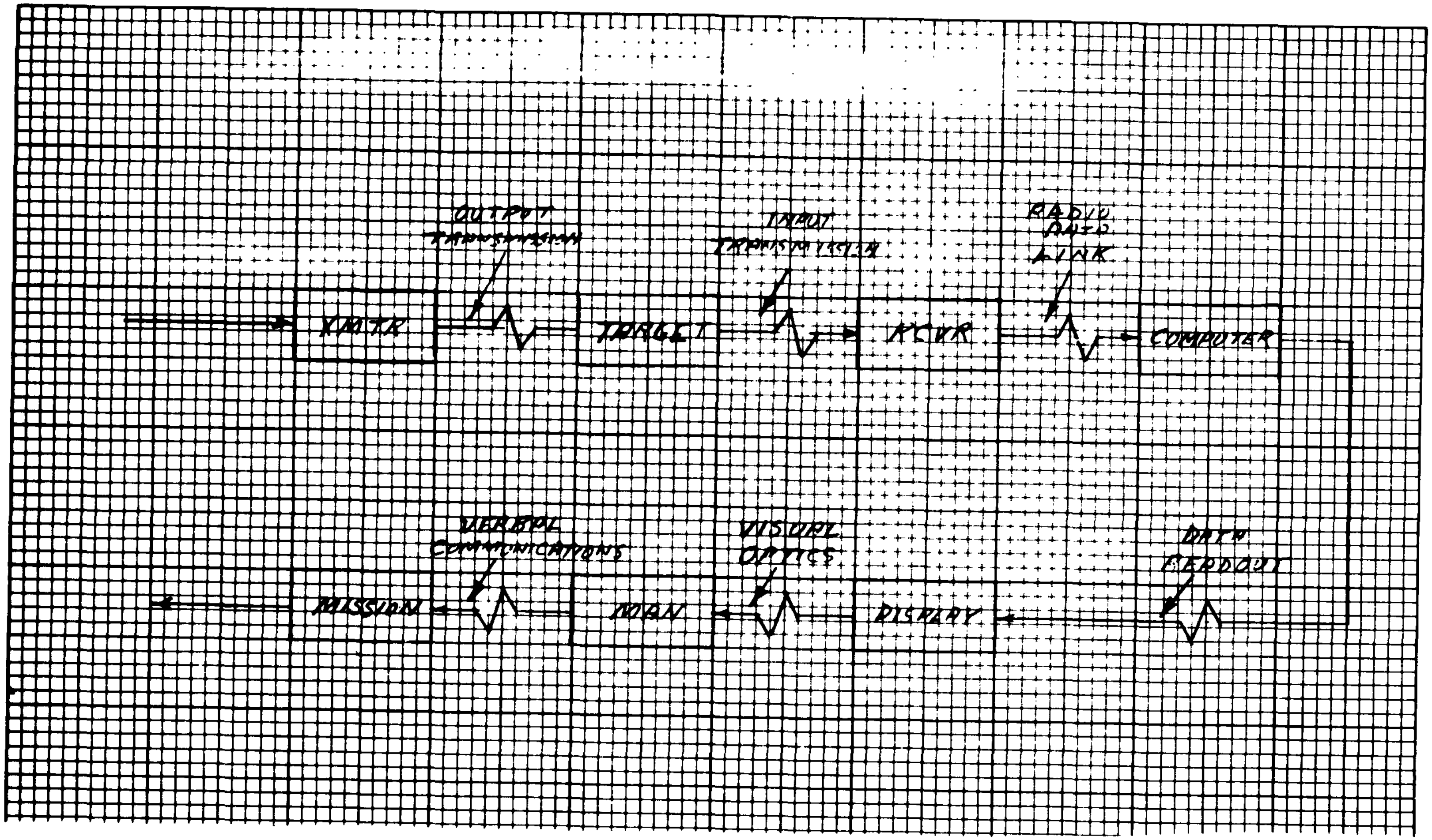

Figure A-1. PERFORMANCE FLOW DIAGRAM (TYPICAL COMMAND AND CONTROL SYSTEM)

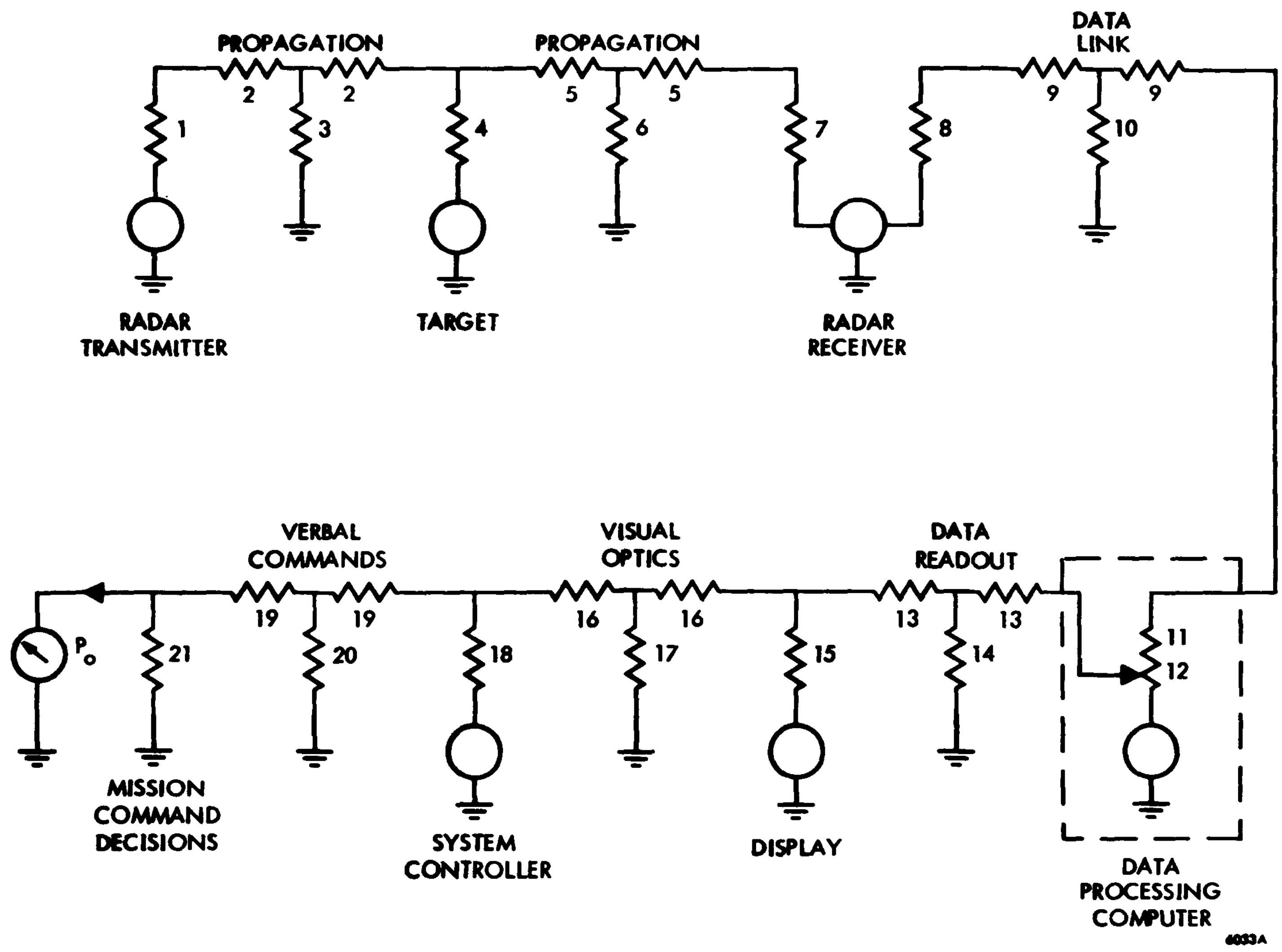

Figure A-2. SPAN MODEL OF AN/FPS-20 COMMAND AND CONTROL SYSTEM

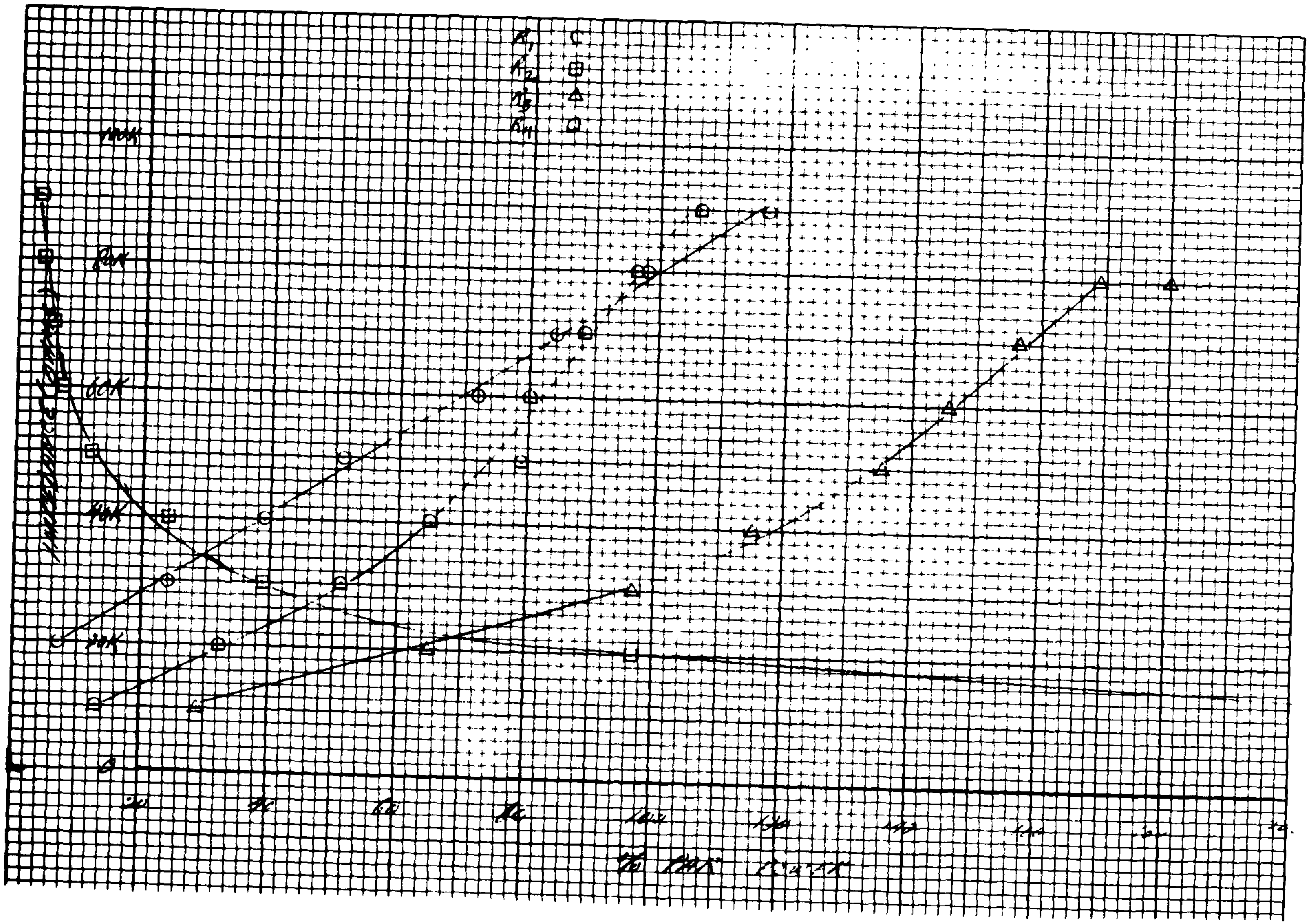

Figure A-3. SPAN SIMULATION SENSITIVITY OF PARAMETERS 1 TO 4

Figure A-4. SPAN SIMULATION SENSITIVITY OF PARAMETERS 5 TO 8

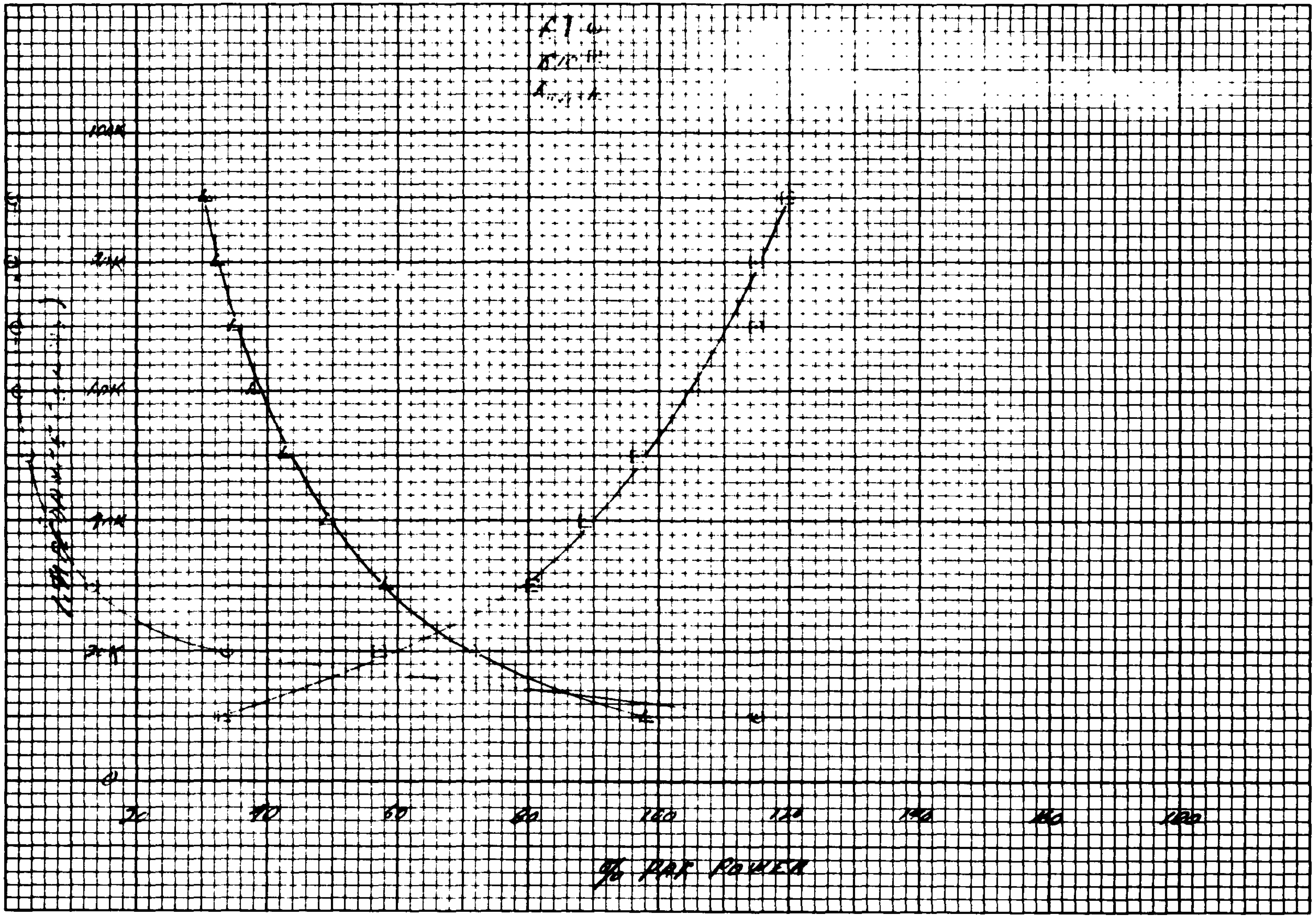

Figure A-5. SPAN SIMULATION SENSITIVITY OF PARAMETERS 9 TO 12

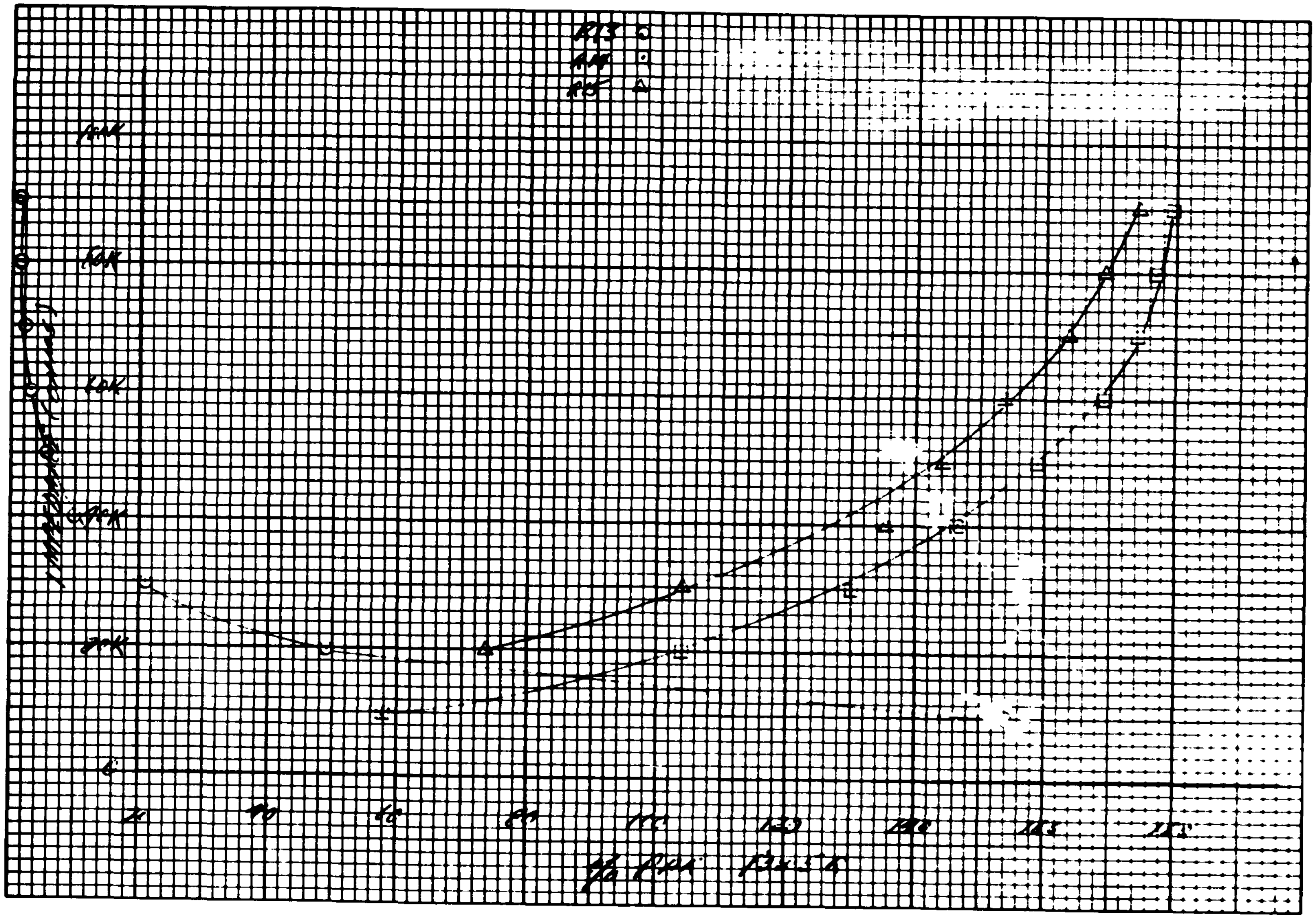

Figure A-6. SPAN SIMULATION SENSITIVITY OF PARAMETERS 13 TO 15

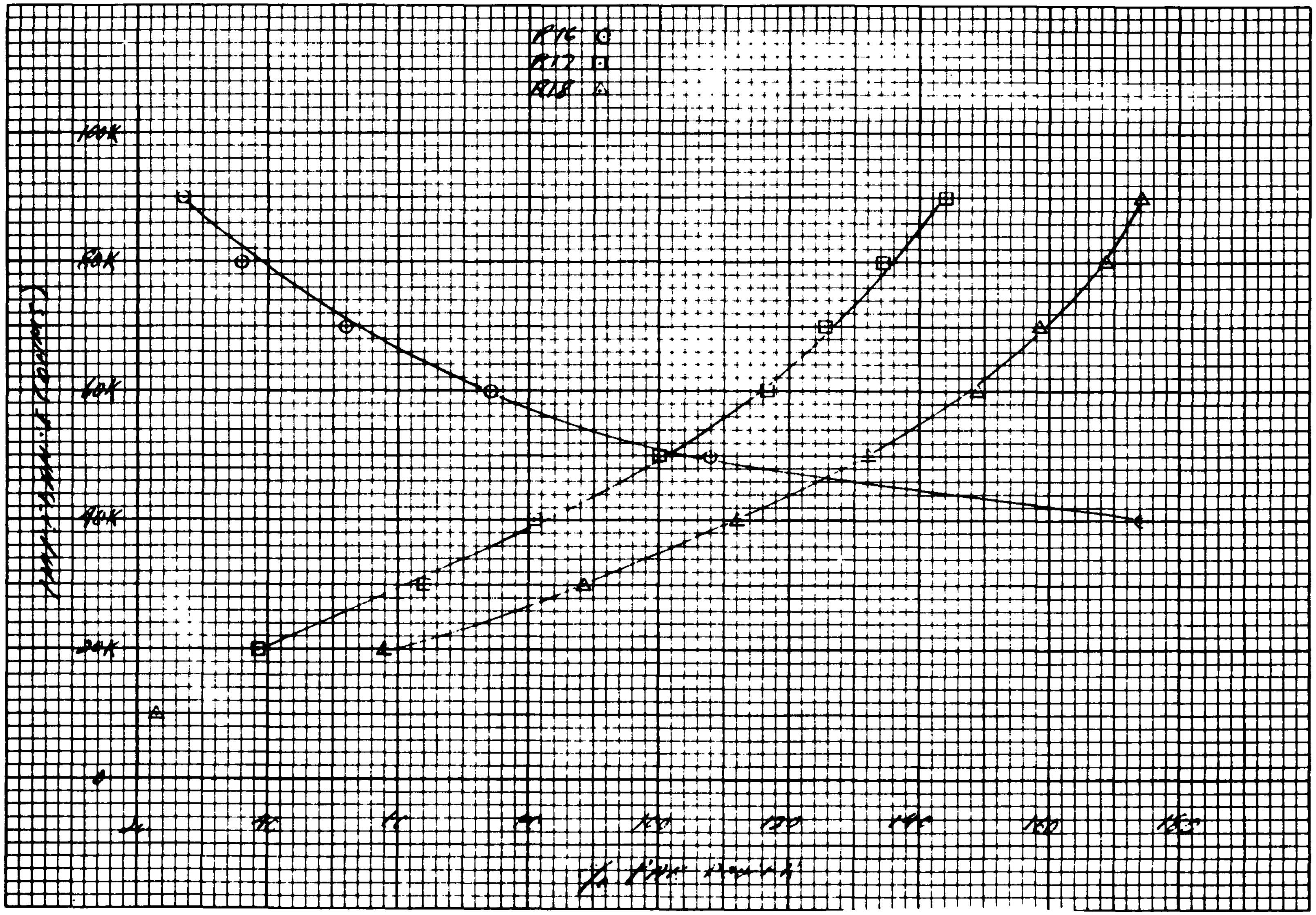

Figure A-7. SPAN SIMULATION SENSITIVITY OF PARAMETERS 16 TO 18

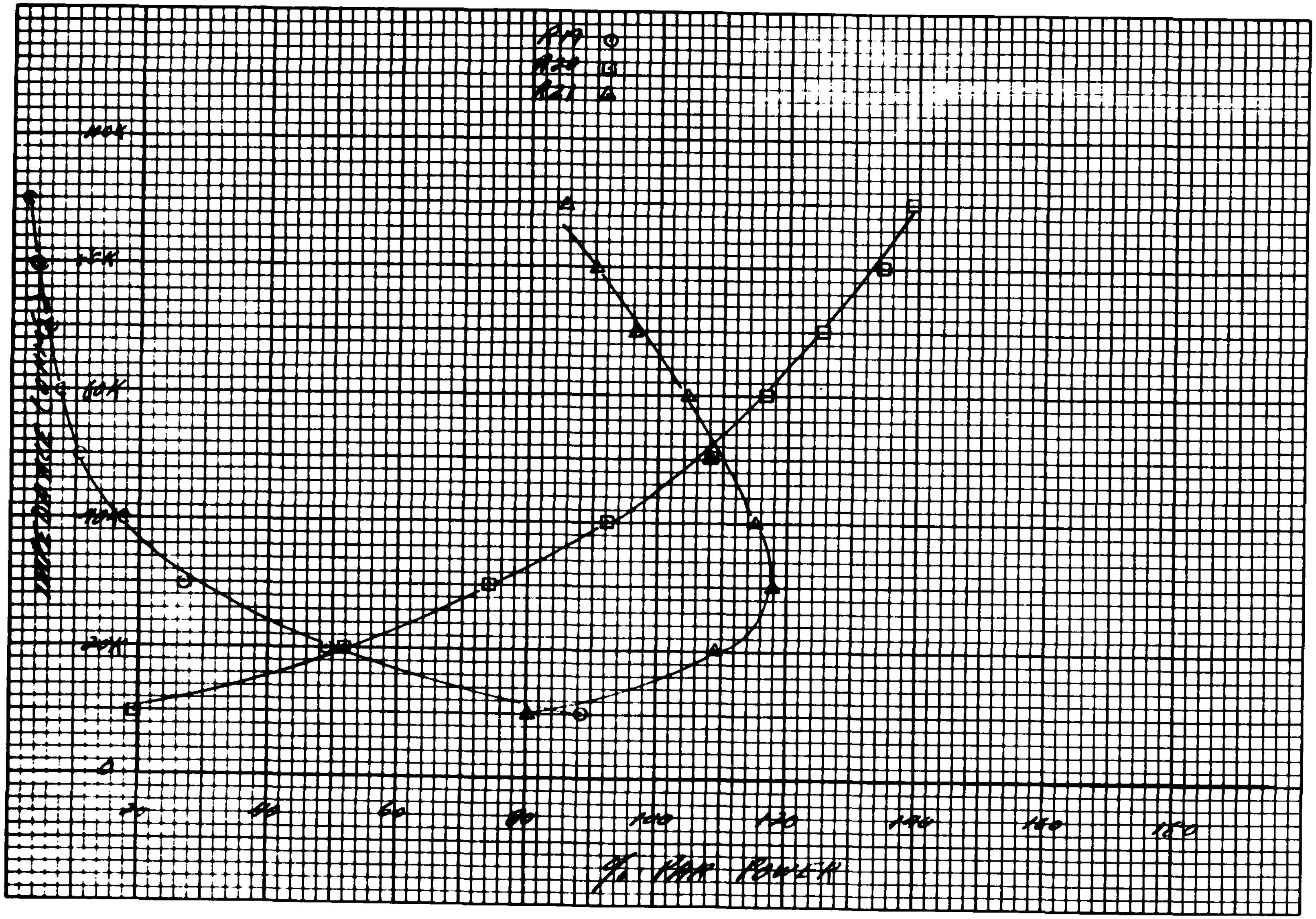

Figure A-8. SPAN SIMULATION SENSITIVITY OF PARAMETERS 19 TO 21

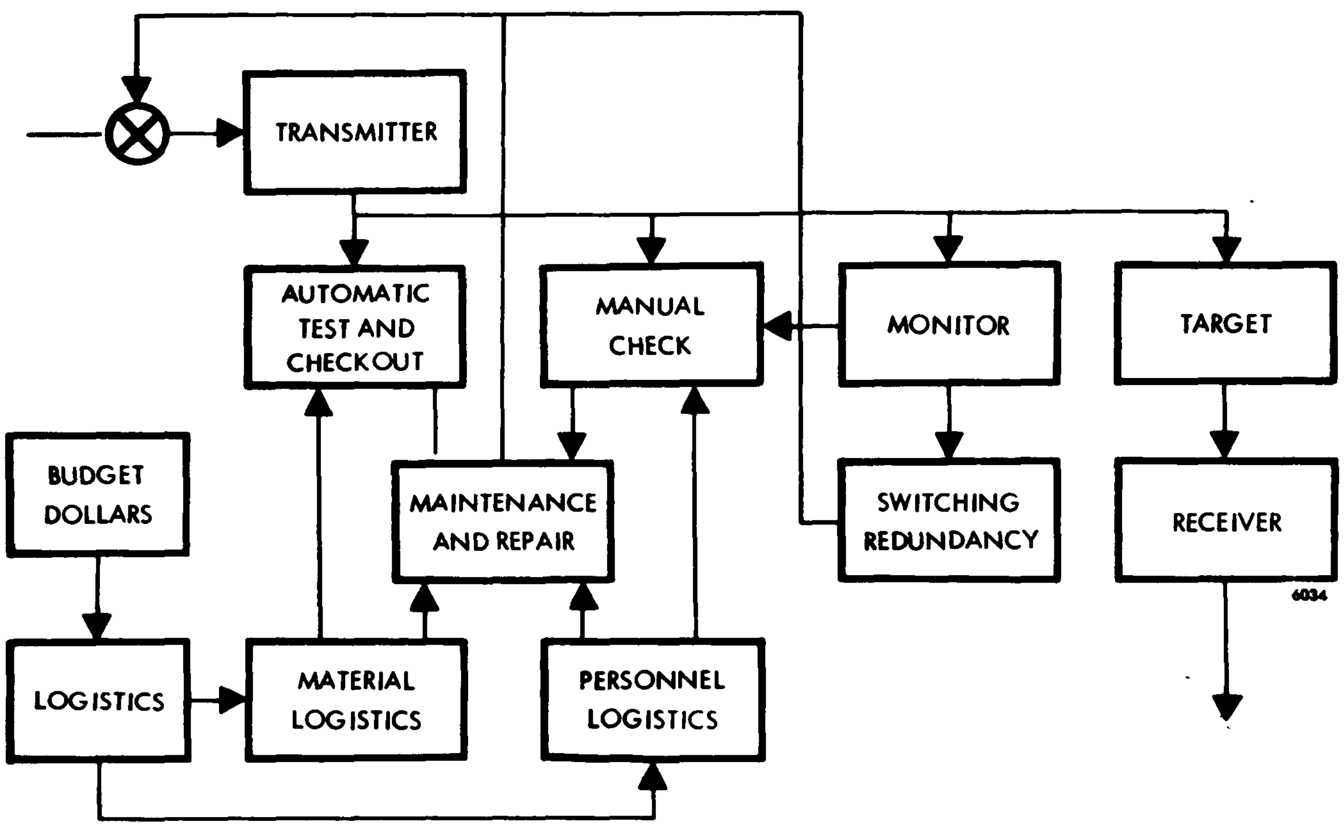

Figure A-9. MAINTENANCE LOOP OF RADAR SYSTEM (FLOW DIAGRAM)

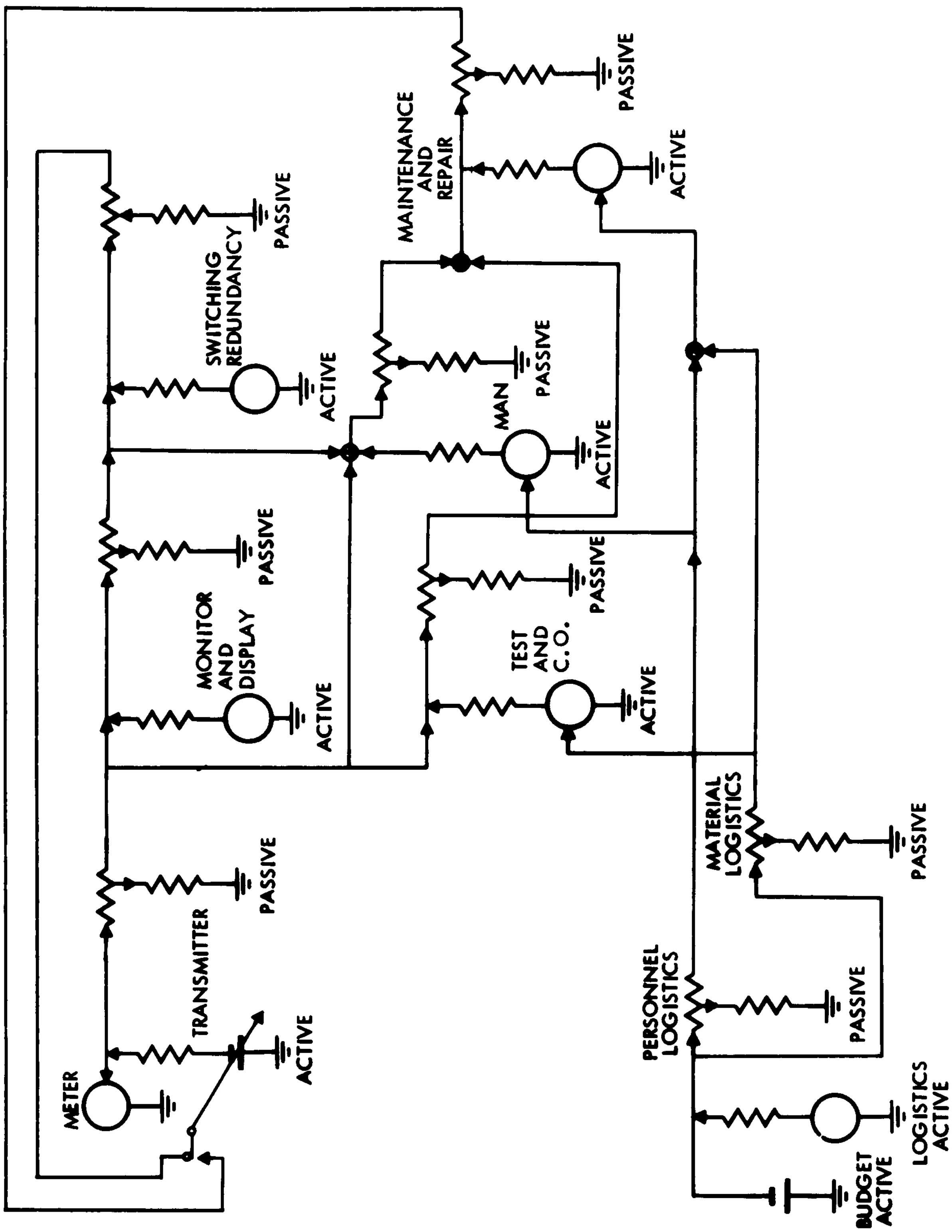

Figure A-10. MAINTENANCE LOOP OF RADAR SYSTEM

For the purpose of definition, reliability is described as the time function of performance quality. Its analog is then the time function of resistance value, which can be modeled by a servo-driven potentiometer programmed to decrease by the $\exp\left(-\frac{t}{m}\right)$ law. If the total effect of a number of unreliable elements in a system must be evaluated simultaneously, the overall probabilities of failure can be obtained by deriving the average information content of the situation, which then becomes part of the effectiveness number.

Maintainability

Maintainability, defined as the function of time flow of maintenance (capability to reverse the degradation in reliability), is then equal to the negative time gradient of reliability. Its electrical analog can be derived by differentiating the reliability resistive analog with respect to time. Intermittent maintenance operations will be analyzed by sampled-data feedback system methods.

Transmission

Transmission phenomena are represented in the "T" resistive networks, in which the values of the series arms represent the "lossy" transmission characteristics of both energy and information, and the values of the shunt arms represent the transfer ratios. Error rates become absorbed in the sum of the common units of performance quality, which are degraded by such factors as attenuation, limited bandwidth, and poor resolution. When errors are due to excessive noise, the effects of noise are simulated by the ground potential level (since noise has zero quality). Raising or lowering this level in any branch of the circuit simulates the variation of noise level in that branch.

Redundancy

The simulation of redundancy at the component level requires a means of representing the method of scheduling the switching of the spare components into place on demand. In effect, this switching is a form of automatic maintenance and can be simulated as such. Then a component with switchable spares becomes a feedback closed loop, with the performance number representing the complexity and reliability of the switching circuit as the feedback function (which often is the dominant function in such circuits). Functional redundancy can be treated in the same manner, since the only difference between components and functional blocks in their performance effectiveness numbers is the numerical value obtained. It is then possible to evaluate trade-off points between functional and component redundancy since the circuit automatically seeks an optimum compromise when feedback is used to optimize this type of performance.

Environment

Environmental effects, since they are essentially due to the flow of energy in some form, in observable patterns and assignable probabilities, are simulated in the same way, and with the same components as performance energy functions, when the patterns can be formulated. When

they cannot, the environments can be treated as low number noise functions representing variable ground levels where they occur. Couplings with environment are modeled in the same way as the transmission channels.

Artificial environments often possess a high degree of correspondence with system behavior. For instance, countermeasures depend heavily on information describing or predicting the performance of the given systems, and are negatively oriented in their directed coupling in order to degrade it. This negative orientation requires a sign-changing or inverting operational amplifier in the analog simulator, with appropriate input couplings from the main system to represent the "spy" channels which give the countermeasures their technical guidance for efficacy in jamming, sabotage, etc. This type of system configuration typical of games situations, takes the forms of an "infinite gain" feedback system in the simulator.

Human Factors

Man-machine compatibility can be evaluated as a special version of impedance matching or balancing, since human performance numbers can be assigned with a useful degree of accuracy. The nonlearning human functions can be represented readily by using an "autotransformer" or inductive potentiometer, with variable settings together with appropriate coupling networks, to model human adaptability. An analog of the self-optimizing capability of human controllers and operators then can be realized by servo-controlling the position of the autotransformer tap to obtain maximum power at some monitorable point, independently of other feedback loops. The learning capacity can be modeled in several ways since it can be represented by a gradual increase in the faculty of human control, as simulated by a servo-controlled resistor which increases according to some known law of learning rate.

Economy

Economic cost factors become weighting factors multiplying the values of the resistors by some coefficients proportional to the relative costs of obtaining equivalent performance in each performance function or parameter. For example, the cost of obtaining 10 db or 3 bits in a radar by increasing the transmitter power 10 times can be prohibitive for already high powered transmitters. On the other hand, the additional 10 db might be obtained less expensively by increasing the size of the receiving antenna or by employing more sophisticated signal-to-noise enhancing circuits, such as masers. Maximum power output of the analog is then obtained at the condition of maximum economy (performance per dollar).

Logistics

Logistics problems can be simulated if the passive performance effectiveness numbers of the materiel are converted to equivalent active performance numbers. Then, in the analog, the logistic channels have as their counterparts the power supplies with known regulation characteristics and with adjustable series resistors for controlling the efficiencies of the flow of electrical

energy, representing materiel. The interrelations which affect different types of materiel in a limited capacity common supply channel are modeled by a constant-current device, such as a transistor.

Eventually, this technique leads to speculation on the possibility of evaluation of the flow of dollars from a limited budget to a given system, and also of the flow of paper communication and authorization through paper channels. While SPAN possibly may be a feasible method for simulating these effects, extreme caution must be used to ensure that the problem is completely isolated from uncontrollable forcing functions which would vitiate the solutions entirely. This condition is very difficult to realize in practice.

Optimization Criteria

Generalized optimization requires an error criterion, or criteria, for improvement, and a means of sensing such errors, interpreting them, and applying the necessary correction measures. Without a thread of common logic uniting these processes, they become quasi-random and their effectiveness is greatly handicapped. The philosophy of SPAN provides a powerful coherent logic for this purpose by postulating that optimum performance consists of maximized effectiveness (described in terms of availability, pattern rank, and information, which are all specialized versions of negentropy). Associating the concept of abstract informational negentropy with concrete energy makes possible a more significant index of system "goodness" in controlling natural forces than has heretofore been proposed. It is indeterminate whether this is the optimum optimizing criterion, but it definitely is one of the criteria employed by the human mind in improving system performance.

Utility and Safety

When the human consumer, considered as a system load, is coupled to any system in SPAN language, the concept of optimum performance coincides with the concept of maximum utility to the consumer. Care must be taken that the degrees of freedom are analyzed separately and not dangerously lumped. The ability to analyze safety requirements also stems from the universal nature of the SPAN metric of human and physical performance, and opens up the possibility of applications to bionics, biometrics, and psychometrics.

Considerable research to reduce this technique to a form suitable for these fields is obviously necessary. However, in the meantime, it appears well suited for such mundane applications as the determination and prediction of the probability of satisfactory operation of a given command and control network, under hypothetical conditions of reliability, human error susceptibility in wartime, equipment casualities, severe environments, and the whole category of troubles which are categorized in "Murphy's law," - "If anything can go wrong, it will." The special virtue of the SPAN methodology is that is offers to temper "Murphy's law", and other superstitious philosophies due to imperfect understanding of our world, by furnishing a little better understanding of it for prediction and prevention of troubles before they occur.

Results

The results of a first optimizing run for the hypothetical command and control system shown in figures A-1 and A-2 are given below:

Condition	R_1	$R_{7,8}$	$R_{11,12}$	R_{15}	R_{18}	R_{21}	Total	Nominal Power Output	Percent
Original Value	64K	71K	34K	34K	40K	50K	293K	0.618W	100
Original Sensitivity	0.15	0.15	0.08	0.16	0.22	0.06			
Final Value	78K	63K	28K	44K	47K	33K	293K	1.61W	260
Final Sensitivity	0.15	0.16	0.11	0.16	0.19	max P_o			

The advantages of matching, as much as possible, the individual sensitivities is shown by the increased power output, at 260 percent of its original value. Since the output power represents the system performance effectiveness or quality, it is readily seen that this increase is obtained with no increase in the total investment in subsystem performance (represented by the total resistance) merely by reapportioning the quality of the performance of each subsystem as represented by its effectiveness number - the value of its resistive analog. In other words we have obtained greater efficiency by rebalancing these values. Greater freedom in matching sensitivities will theoretically enable even more spectacular gains. Additional increases might be obtainable by similarly varying the values representing the coupling channels between subsystems.

If these values had been weighted by their respective cost factors (performance per dollar), the increase would represent an equivalent increase in system performance per dollar. If desired, some of this increased performance capability can be allocated to assure greater reliability for a given mission, or greater meantime between failures, a lighter maintenance schedule, a greater safety margin, a more demanding mission, greater transmission capacity, etc.

These changes, translated back into real system parameters, give the following recommendations:

(1) Increase the effectiveness of the radar transmitter by 20 db (7 bits)
(2) Decrease the receiver effectiveness by 12 db (4 bits)
(3) Decrease the effective data handling capacity of the computer by 3 bits (per second)
(4) Increase the capacity of the display by 5 bits (per second)
(5) Increase the capability of the controller by 10 db (3.3 bits)
(6) Decrease the complexity of the command/mission by 8.5 bits (decisions) per second.

Since this optimization has been made with respect to performance effectiveness only, and does not include cost factors, it is obvious that some of these recommendations may not be economically sound. For example, the increase in radar transmitter capabilities of 20 db will not be cheaply obtained. If the parameters modeled had been performance times dollars, a different

equilibrium set of values would have been determined. In spite of this, it is apparent that, if cost is no object, the system can be greatly improved by these expedients. Even if the transmitter change is not made, an impressive increase in performance effectiveness can be achieved through the remaining revisions. The final decision as to the form of these changes reverts to the system designer, who has several options at his disposal.

The net effect of this retrofitting of system organization is greater internal compatibility between subsystems. It also can be called a process of system integration. Where the probability of potential mismatching is high, as in man-machine combinations, the improvement factors realized can be even greater than that shown in the example given. In effect, the system performance mathematics has been translated into a logical language in which the circuit analog can do the thinking. It is a restricted form of artificial intelligence machine which operates according to the logic of physical energy causality. Since this agrees with the logic of the majority of physical and engineering systems, it is capable of acting as an accurate model of any such system in this language.

The reverse of this optimizing process can be thought of as that which a system undergoes with degradation of performance, commonly called reliability degradation. The interaction of individual reliabilities has not received much analytical consideration in most system studies, but the SPAN philosophy postulates that it exists and plays an important part in the degradation of behavior of complex systems. When it is described as a time function, it produces the familiar reliability expressions for mean time to failure, mean time between failure, etc. Putting these into the logarithmic form of SPAN parameters makes them very much more tractable for analytical purposes. They can be simulated in the same way as the optimizing process described above, and their effects can be evaluated. The resulting degradation of system effectiveness can then be predicted with great accuracy, subject to the initial accuracy of the input information.

Maintenance Loop

The simulation of the maintenance loop of a typical portion of the command and control system described in the preceding section, is shown in figures A-9 and A-10. Figure A-9 is a block diagram of the maintenance associated with the radar transmitter of the AN/FPS-20 radar, showing essentially three types of maintenance - the automatic or redundancy-switching, the completely manual, and the combination of automatic test and checkout equipment with manual repair. Included also in the model are the logistic channels of supply of repair parts and materiel, and skilled personnel. These are shown as dependent on a fixed budget for expenditures.

Tentative quantitative effectiveness numbers have been assigned where firm information has not been readily available. Since this is an exercise rather than an evaluation of an existing real system, the purpose of demonstrating the practical application of the method is served by the adoption of artificial values just as well as by the use of real values from a current system.

The effectiveness numbers employed are based on the following assigned characteristics:

Functions	Specification	Effectiveness Number
Radar transmitter		
Same as in basic system		32 bits
Transmission to monitor		
Attenuation	42 db	14 bits
Losses in reliability, resolution, accuracy	3 db	1 bit
Pattern of sampling function	10 degrees of freedom	10 bits
Monitor and display		
Pattern of response	6 degrees of freedom	6 bits
Transmission to redundancy switch		
Losses (reliability, degradation, energy loss)	15 db	5 bits
(Resolution, energy level, reliability, BW, etc., of comparator)	9 degrees of freedom	9 bits
Redundancy switch		
Switching power, number of contacts	5 degrees of freedom	5 bits
Transmission to repair loop		
Attenuation (contact reliability)	2 degrees of freedom	4 bits
Transfer (functions, power level)	6 degrees of freedom	6 bits
Test and checkout equipment		
(functions, BW, power level, reliability)	20 degrees of freedom	20 bits
Transmission to maintenance and repair		
Losses (information, significance, level, intelligibility)	6 degrees of freedom	6 bits
Transfer effectiveness (pattern of instructions, accuracy, level, BW)	10 degrees of freedom	10 bits
Human maintainer		
Same as in basic system		10 bits

Functions	Specification	Effectiveness Number
Transmission from man (observer) to maintenance and repair		
Attenuation (forgetfulness, confusion, errors, energy loss)	4 degrees of freedom	4 bits
Transfer effectiveness (complexity of repair function, energy level, BW, reliability)	10 degrees of freedom	10 bits
Maintenance and repair		
(Complexity of repair function, energy level, BW, reliability, accuracy)	20 degrees of freedom	20 bits
Transmission of maintenance and repair effects to transmitter performance		
Attenuation (faulty workmanship, etc.)	1 degree of freedom	1 bit
Transfer (goodness of repair, BW, accuracy, energy level, reliability)	19 degrees of freedom	19 bits
Logistics function		
(Passive effectiveness numbers per unit time)		0.1 bit per second
Logistics transmission, personnel		
Attenuation (degradation in personnel availability, SNAFU factor)		1 bit
Transfer (IQ level of personnel, complexity of training and assignment, handling efficiency)		10 bits
Logistics transmission, materiel		
Attenuation (losses, damage, errors)	4 degrees of freedom	4 bits
Transfer (degrees of freedom, reliability, BW, equipment complexity)	20 degrees of freedom	20 bits

Complete simulation of this loop requires a servodrive for R_1, the transmitter analog. A comprehensive realistic optimization treatment of this system will result in preferred values for the various parameters, weighted by relative cost factors, which were not available at the time of this report.

The simulation equipment used in this study is shown in the photograph (figure A-11). The black cylindrical objects are dual 10-turn potentiometers, which can be servo-driven from beneath, when plugged into a servomotor specially packaged for the purpose. The gold-colored cylindrical objects are single 10-turn potentiometers, which also can be servo-driven in the same way. The two plug-in operational amplifiers can be discerned in the center and right-hand portions of the six-panel circuit designer plug-board assembly. Direct-reading dials on each potentiometer package permit quick determination of the resistance values. The 48 black knobs control 48 auxiliary potentiometers for fixed settings. The circuit designer plugboards provide 1200 circuit connection jacks, and 18 octal sockets, into which are plugged the servomotors, operational amplifiers and polarity-reversing relays.

The simplicity of this arrangement should be contrasted with the complexity of the programming for an equivalent treatment on a high-capacity digital computer, to obtain a basis of comparison between the two methods. The accuracies obtainable by the simulation equipment shown is at least of the same order of magnitude as that of conventional data on the behavior of existing systems. The time required to program, setup, run and evaluate the problems described, using manual approximation rather than self-maximizing servo techniques (since the servo components had not been delivered in time for the conclusion of this report) was approximately 40 man-hours. More detailed and exhaustive treatments would take correspondingly longer to analyze. The use of self-maximizing servodrives with appropriate feedback criteria would reduce the time for solution to one-tenth or less, of that taken by manual manipulation. The demonstration of this capability can possibly be a suitable task for Phase II of this program.

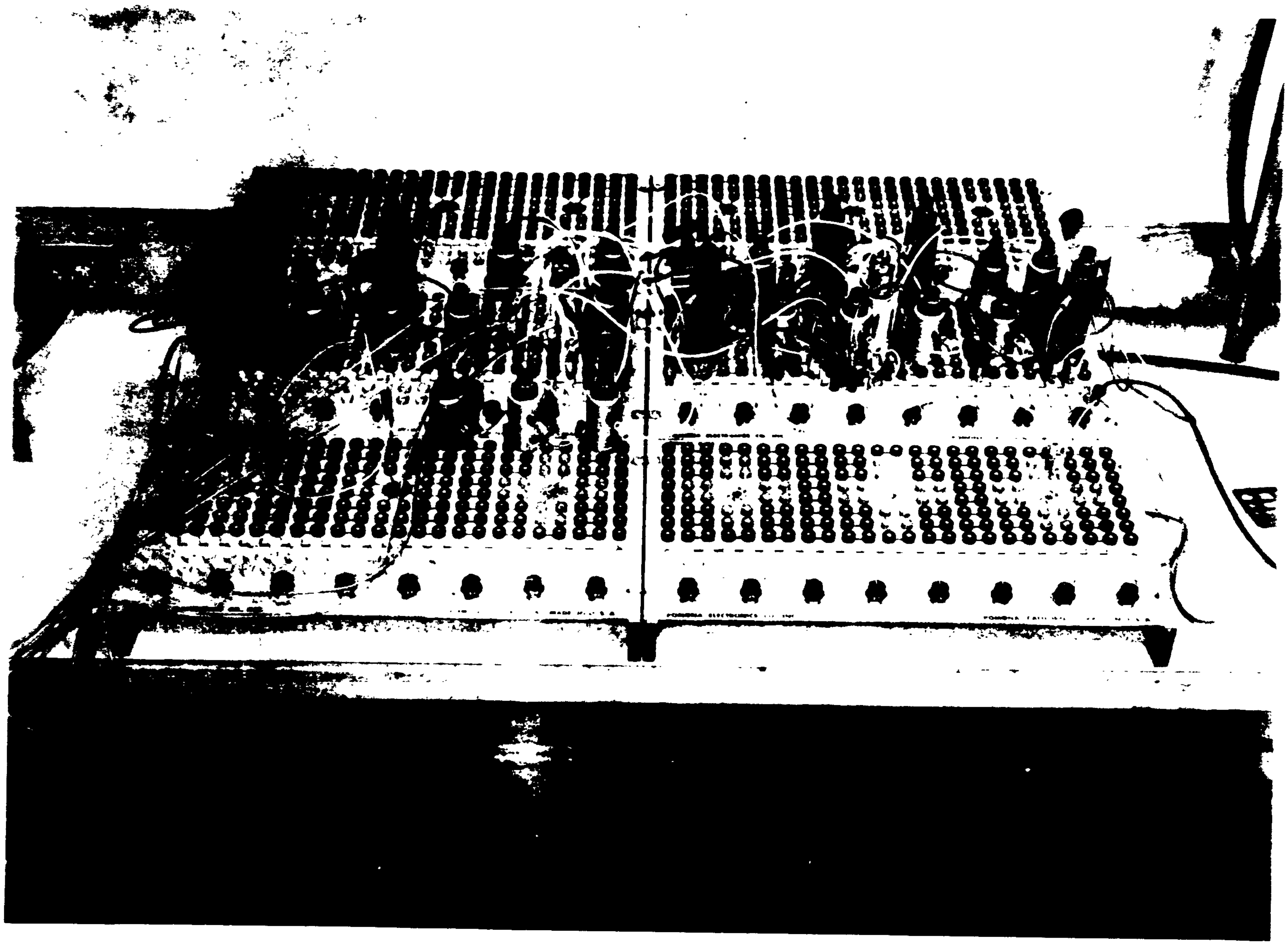

Figure A-11. SIMULATION EQUIPMENT

APPENDIX II
SPAN FROM A LAYMAN'S POINT OF VIEW
by H. N. Sissel

This appendix has been prepared to assist personnel, who are not familiar with information theory and related fields, to better understand the main body of this report. The approach taken for this explanation was to have an engineer, also unfamiliar with the subject, to list some questions, pertaining to the analysis of a system by SPAN, that might come up as the report is being read. He was also asked to present some simplified examples. These questions and examples, along with applicable answers, are presented below to help answer, or suggest answers to questions the readers of this report may have.

Question 1: What is the basic problem in understanding and applying SPAN?

Answer 1: It seems to be a matter of communication-indoctrination. A background in probability theory, information theory, and thermodynamics, and a grasp of electronics is required. There may not be so much a question of understanding, for the basic idea is fairly clear, but rather one of applying SPAN techniques. This seems to require an intuitive grasp of the system analogy.

Question 2: What is conveyed by this report?

Answer 2: SPAN seems to be a means of converting engineering and other units into a common language.

Question 3: Answer 2 is all well and good, but what is gained by it?

Answer 3: A greater facility in analysis of complex systems.

Question 4: How is it applied?

Answer 4: By constructing first a mathematical model, and, second, a physical model, of the given system.

Question 5: What are the criteria for setting up the analog in a system?

Answer 5: The criteria are:

(1) The degree of detailed breakdown into black boxes.

(2) The need for information on interaction between black boxes, and with environments, including mission.

(3) The kind of maximization or optimization required.

(4) The degree of identification which must be carried through the solution.

Question 6: How is it determined which elements are to be included in the analog?

Answer 6: The elements in the analog simulation are:

(1) The analogs of the black boxes

(2) The analogs of the independent energy or information sources

(3) The analogs of the transfer channels between black boxes.

Question 7: How is the complexity number, pattern rank, transfer function, etc, determined?

Answer 7: Determination of these numbers is described in tables A-1 and A-2.

One means of attacking this problem of indoctrination is to make a list of some of the questions which arise when trying to use SPAN.

Some common questions about using SPAN (assuming a system is to be analyzed via SPAN) now follow.

Question 8: Why should SPAN be used?

Answer 8: To obtain a better organized, simplified model of complex system behavior.

Question 9: What is there about SPAN that makes it better than existing philosophies?

Answer 9: Greater logical coherence, greater unifying scope, greater compatibility with modeling methods.

Question 10: To what systems should SPAN be applied?

Answer 10: Physical and engineering systems, including human operators, controllers and observers.

Question 11: Since there are many systems which SPAN may be applied to, are any systems ideal for the utilization of SPAN?

Answer 11: Those which suffer from over-complexity, unreliability, or other acute, difficult-to-solve system idiosyncrasies.

Question 12: What properties do these systems have which make them ideal for the use of SPAN?

Answer 12: System indigestion, unbalance, inefficiency, incompatibility or heterogeneity.

Question 13: How is the system broken down into SPAN units?

Answer 13: By drawing a functional block diagram and converting the real parameters in table A-1 to SPAN parameters for each block.

Question 14: How much of the system can be broken into its component parts before reaching the point of diminishing returns?

Answer 14: This depends on practical importance of detailed information.

Question 15: What is the criterion for this choice?

Answer 15: Cost of the analysis.

Question 16: What is the philosophy of breakdown (analysis) of the system into SPAN units?

Answer 16: The degree of interest in details determines the complexity which is lumped in a particular black box.

For this discussion a simplified version has been prepared of one of the previous tables used to convert the real system of units to SPAN language.

Table A-1. SPAN CONVERSION LANGUAGE

REAL		SPAN		ANALOG
Energy e		$\log_2 e$		O Reference
times		plus		plus
Scale K		$\log_2 K$		A Availability
times		plus		plus
Dimensional BW x $A^{\alpha\beta \ldots n.}_{\gamma\delta}$	=	$\sum_1^n \log BW$	=	ρ Pattern
times		plus		plus
Probability p		$-\log_2 p$		H Information
equals		equals		equals
System efficiency = $e \times K \times BW \times A^{\alpha\beta \ldots n}_{\gamma\delta} \times p$		ξ = Effectiveness		$\xi = A + \rho - H$

$\xi = \overline{qV}$ = volt-coulombs

Performance = $\frac{d\xi}{dT} = \overline{IV} = \overline{P}$ = Power.

From this table it can be seen that for each element of the system three items must be known to determine the performance of the elements of the system - the scale factor, K; the probability, p; and the dimension. It becomes obvious that once a reference energy level has been arrived at, it is possible to determine the degree that the elements are above this level and thereby determine the scale factor, K. The availability, A, is determined by taking the log to the base two of K. This is a fairly obvious number since it usually is possible to determine the total energy a given element in the system is capable of producing.

It is a little more difficult to assign meaning and numerical values to a determination of the dimension and the probability. Probability will be considered first. The probability will depend on the uncertainty in the function of the element. In one case, the accuracy of measurement of the element might be given or, in another, the percent of the charge of a capacitor leaking away

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REAL		SPAN		ANALOG
Energy e		$\log_2 e$		O Reference
times		plus		plus
Scale K		$\log_2 K$		A Availability
times		plus		plus
Dimensional $BW \times A_{\gamma\delta}^{\alpha\beta \ldots n.}$	=	$\sum_1^n \log BW$	=	ρ Pattern
times		plus		plus
Probability p		$-\log_2 p$		H Information
equals		equals		equals
System efficiency = $e \times K \times BW \times A_{\gamma\delta}^{\alpha\beta \ldots n} \times p$		ξ = Effectiveness		$\xi = A + \rho - H$

$\xi = \overline{qV} =$ volt-coulombs

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in a given time. The choice of probability will depend to a great extent upon the person making the choice; however, the choices made by different people should have a fairly close correspondence. Once a reasonable estimate for the probability has been determined, a value for information is arrived at by taking the log to the base two of the probability.

The final item to be determined is the dimension. In many ways this quantity is the most nebulous and hard to define. It has different names like resolving power or bandwidth which mean different things to different people.

It usually consists of both a scalar and a vector character. The scalar portion is the measure of extent, over a given range, with a given resolution (minimum increment). The ratio of these values is the "bandwidth." The pattern rank includes its logarithm (base two). The "n" orthogonal unit vector components are included as "n". Hence pattern number $\rho_c = \sum_1^n \log_2 BW$.

Taking the log to the base two of this number gives the pattern of the element. Effectiveness is determined by adding together availability and pattern, and subtracting information. The value of performance is found by taking the derivative of effectiveness with respect to time.

From these numbers an analog of the system can be set up on a system analyzer and the values which produce the highest performance can be found.

To end this discussion, a simple system will be used as an example and SPAN will be applied to it. The system chosen is the actuation of a control stick.

From the above discussion it is evident that a reference level for the energy must be established. In this case, since the motion is rotational, a reference energy of 1 inch-pound will be selected. The total energy of the system is 8 inch-pounds. From this it can be seen that the scale factor is 8. Taking the $\log_2 8$, the electrical analog of availability is 3 volt-coulombs. The range of the control stick is $\pm 16°$ and the resolution which can be obtained by the stick is $1°$. Establishing the bandwidth as equal to the range divided by the resolution gives a bandwidth equal to 32. Taking $\log_2 32$, gives a pattern number of 5 and a corresponding analog quantity of 5 volt-coulombs.

The system has an accuracy of 97 percent which gives a probability of 0.97. The amount of information derived from this probability is +0.044 volt-coulombs. By taking the sum of the availability and the pattern number and subtracting the information, a value for the effectiveness of 7.956 volt-coulombs is arrived at. Since the performance is the time derivative of the effectiveness, the performance is then 3.978 volt-coulombs per second. A more complete description may be seen in table A-2.

Table A-2. TYPICAL CONVERSION OF CONTROL STICK ACTUATION

ENGINEERING UNITS	TRANSFORMS: LINEAR	TRANSFORMS: LOGARITHMIC	SPAN ANALOG UNITS
ENERGY (TORQUE)			
Ref. level, 1 in.-lb	E = 1 in.-lb	$\log_2 E_r$ = 0 bits	
Energy level, 8 in.-lb	Scale Factor K = 8	$\log_2 K$ = 3 bits = A	A --- 3 volt-coulombs
Response time, 2 sec	T = 2	$\log_2 T = 1$	
Power 4 in.-lb/sec	$\frac{dK}{dT} = 4$	$\log_2 \frac{dK}{dT}$ = 2 bits = $\frac{dA}{dT}$	$\frac{dA}{dT}$ --- 2 volt-coulombs/sec
Pattern 1-D (Yaw ∠)			
Range, ±16°			
Resolution, 1°			
Range/Resolution, 32	Resolving Power (Bandwidth) BW = 32	$\log_2 BW$ = 5 bits = ρ_c	ρ_c = 5 volt-coulombs
Torque Ref. Level, 1 in.-lb/°/sec			
Precision, 0.9°			
Error, 3 percent			
Accuracy, 97 percent	p = 0.97	$-\log_2 p$ = 0.044 bits = H	H = 0.044 volt-coulombs
Total Effectiveness 8 in.-lb/sec | +16° ±0.45°, -16° ±0.45°	$E \cdot \eta = 8 \times 32 \times 0.97$	$\xi = A + \rho_c - H = 7.956$	ξ = 7.956 volt-coulombs
Performance $\frac{8}{2}$ in.-lb/sec | +16° ±0.45°, -16° ±0.45°	$\frac{d}{dT}(E \cdot \eta) = 4 \times 32 \times 0.97$	$\frac{d\xi}{dT}$ = 3.956 bits/sec	$\frac{d\xi}{dT}$ = 3.956 volt-amperes

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